Financial Conditions and the Risks to Economic Growth in the United States Since 1875*

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Abstract

We explore the historical relationship between financial conditions and real economic growth for quarterly U.S. data from 1875 to 2017 with a flexible empirical copula modelling methodology. We utilize variants with both linear and non-linear dependence, and with both Gaussian and non-Gaussian marginal distributions. Our results indicate strong statistical support for models that are both non-Gaussian and non-linear for all sub-samples of historical data, with considerable heterogeneity across sub-samples. We demonstrate that ignoring the contribution of financial conditions typically understates the conditional risks to economic growth. For example, accounting for financial conditions more than doubles the probability of negative growth in the year following the 1929 stock market crash.

Keywords: Probabilities of economic events; Vulnerable growth; Growth at risk; Great Depression.

JEL codes: C14; C32; C53; E37; E44; N10; N20

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1 Introduction

Some important recent empirical macroeconomic research highlights the impact of financial conditions on the risks to economic growth. Papers by, among others, Adrian, Boyarchenko and Giannone (2019a, ABG) and Adrian et al (2018) argue that economic growth has a non-Gaussian distribution. Their research quantifies the vulnerability of economic growth using quantile regression methods and provides the basis for the IMF bilateral surveillance tool, "growth at risk".

Earlier studies finding strong empirical linkages between the severity of recessions and financial crisis using historical data include, for example, Bordo and Haubrich (2010), Schularick and Taylor (2012), Jordà, Schularick and Taylor (2013, 2016) and Jalil (2015). These papers build on a considerable volume of historical qualitative analysis emphasizing the complexity of economic growth during episodes of financial crisis.

In this paper, we take a broad historical perspective of the relationship between economic growth and financial conditions using quarterly U.S. data from 1875 to 2017. Given the emphasis in the historical literature on the complexity of macroeconomic growth, we compute probabilistic assessments utilizing a copula modelling strategy. Our methodology has sufficient flexibility to encompass linear and non-linear variants, and is appropriate for both Gaussian and non-Gaussian distributed macroeconomic variables.

Looking across various sub-samples of historical U.S. data and across a variety of measures of financial conditions and economic growth, we find strong evidence that both macroeconomic variables are individually non-Gaussian distributed, with non-linear dependence between them. Based on Root Mean Squared Error and a KLIC-based measure of probabilistic predictive content, we find financial conditions matter for economic growth. Restricting attention to linear dependence limits the predictive content from financial conditions, regardless of whether or not the fitted marginal distributions are Gaussian. Limiting attention to Gaussian marginals, financial conditions have substantial predictive content provided dependence is non-linear. These results hold regardless of the sub-sample considered, even though there is considerable heterogeneity across sub-samples.

We consider the impact of financial conditions on the probabilities of various economic growth events based on our preferred non-Gaussian and non-linear copula specification. We find that ignoring the contribution of financial conditions typically understates the conditional risks to economic growth. For example, looking at the year following the 1929 stock market crash, including financial conditions more than doubles the risk of negative economic growth from 31% to 79%.

In terms of our modelling methodology, we build on recent time series research adopting copula methods to separate margins from dependence. Following, among others, Smith (2015), Smith and Vahey (2016), Loaiza-Maya and Smith (2019) and Karagedikli, Vahey and Wakerly (2019), we fit non-parametric marginal distributions based on the Empirical Cumulative Distribution Function (ECDF) for each macroeconomic variable. In this manner, we allow scope for non-Gaussianity in the macroeconomic variables, while modelling both cross-sectional and serial dependence in the copula density.

In contrast to these macroeconomic copula studies, we deploy an empirical copula approach, fitting the dependence parameters on pseudo data (derived from the non-parametric marginals). Specifically, we deploy a kernel density estimator, with adjustments for boundaries, to estimate the dependence non-parametrically on the unit cube. Direct non-parametric estimation of the joint distribution based on the observed time series—rather than pseudo data—combines any non-Gaussianity and non-linearity; DiNardo and Tobias (2001) and the density impulse responses of Adrian, Boyarchenko and Giannone (2019b) provide examples of this approach.

In addition to ABG (2019a), Chavleishvili and Manganelli (2019), Ferrara, Mogliani, and Sahuc (2019) and Loria, Matthes and Zhang (2019) all adopt (modified) quantile regression methods to study the importance of non-Gaussianity for economic growth with modern era data. Bernard and Czada (2015) discuss the non-robustness of the (locally) linear dependence estimates delivered by quantile regression methods in the presence of non-Gaussianity. Amengual, Sentana and Tian (2019) propose robust methods to compute linear dependence with copulas. (We adapt their approach to estimate a specification with linear dependence and non-Gaussian distributed variables.) Carriero, Clark and Marcellino (2019) note that the modern era U.S. data contain relatively few pertinent observations to assess tail behaviour for output growth. Looking at historical data gives greater scope for studying predictability for tail economic growth events.

In summary, by modelling non-linearity and non-Gaussianity separately, in this study we clarify the complex historical relationship between financial conditions and economic growth. Both features matter for probabilistic economic growth assessments based on financial conditions with historical data.

The remainder of this paper is as follows. In Section 2, we set out our macroeconomic copula modelling methodology, describing how the approach has sufficient flexibility to separate non-Gaussian and non-linear features for the model space explored by ABG (2019a). In Section 3, we summarize the historical data used, exploring variations in the measures of economic growth and financial conditions by sub-sample. For the modern era data, we also consider the same National Financial Conditions Index (NFCI) used by ABG (2019a), a factor model based index of over 100 indicators of financial conditions, in addition to other measures of financial conditions. In Section 4, we present our prediction results, probabilistic assessments of the upside and downside risks to economic growth and the implications of our results for the relationship between financial conditions and economic growth. In the final section, we conclude by discussing briefly some future avenues for research utilizing non-Gaussian and non-linear models of economic growth.

2 Macroeconomic Copula Modelling

By utilizing empirical copula modelling methods, we can distinguish between non-Gaussianity and non-linearity in the sample data—in contrast to quantile regression based approaches. A copula model separates the margins from the dependence structure, facilitating estimation of a robust, distribution-free measure of dependence based on ranks.

In this section, first we describe how the copula modelling methodology separates margins from dependence and then we give a detailed account of our approach to fitting the empirical copula models to macroeconomic data. We conclude this section by describing prediction and evaluation.

2.1 Separation of Margins and Dependence

To show how copulas help distinguish between non-Gaussian and non-linear features in general, we begin by exploiting Skars' Theorem (Nelsen, 2006) to decompose a joint distribution into univariate marginal distributions and the dependence structure.

Suppose we consider a multivariate joint distribution for S variables contained in $\boldsymbol{Z}$. Sklar's Theorem indicates that a copula exists, C, which separates the joint distribution into the univariate margins and the dependence structure. The copula function is unique under certain regularity conditions but unknown and has to be fitted in applications.

From Sklar's Theorem we express the joint distribution function for $\boldsymbol{Z}$ using the copula function, C:

$$F(\boldsymbol{z}) = C(\boldsymbol{u}) \tag{1}$$

where $\boldsymbol{z} = (\boldsymbol{z}_1', \ldots, \boldsymbol{z}_S')'$, $\boldsymbol{u} = (\boldsymbol{u}_1', \ldots, \boldsymbol{u}_S')'$. The joint distribution on the right hand side, C, is defined on the S-dimensional unit cube, $[0,1]^S$. The S variables in $\boldsymbol{u}$ are often referred to as "copula data". Denoting the Cumulative Distribution Function, $F(\cdot)$, then for each variable we can define the copula data as $\boldsymbol{u}_s = F_s(\boldsymbol{z}_s)$ so that they are individually uniformly distributed for each margin (variable).

We differentiate the distribution function, equation (1), to give the probability density of $\boldsymbol{Z}$ as the product of the copula density and the S marginal densities:

$$f(\boldsymbol{z}) = c(\boldsymbol{u}) \prod_{s=1}^{S} f_s(\boldsymbol{z}_s) \tag{2}$$

where $f_s(\boldsymbol{z}_s)$ is the marginal density of $\boldsymbol{z}_s$ and $c(\boldsymbol{u}) = \frac{\partial}{\partial \boldsymbol{u}} C(\mathbf{u})$ is the copula density.

Hence, the copula density captures the dependence and is separated from the S marginal densities. In this framework, non-linearity is a property of the dependence, whereas non-Gaussianity is captured by the margins.

2.2 Fitting Empirical Copulas to Macroeconomic Variables

As noted above, although the copula methodology seperates margins from dependence, the copula function has to be fitted in applied work. In our time series setting, each of the S variables contains a time series of length T and the corresponding copula data are $\boldsymbol{u}_s = (u_{s,1}, \ldots, u_{s,T})'$. Throughout our application of copula modelling to macroeconomic variables, we assume that data are stationary; see Smith (2015) for a discussion of stationarity issues for copula modelling with macroeconomic time series.

For each variable, the Probability Integral Transforms (PITS) provide feasible copula data, derived in a computationally convenient manner from the Empirical Cumulative Distribution Function (ECDF). Previous applications working with ECDF marginal distributions on modern macroeconomic data include Smith and Vahey (2016), Loaiza-Maya and Smith (2019) and Karagedikli, Vahey and Wakerly (2019).

Whereas those macroeconomic copula modelling papers fit parametric copula densities, in this paper we adopt a more flexible route by adapting empirical copula methods. Empirical copula models have both non-parametric marginals and a non-parametrically estimated copula density; see Deheuvals (1979), Deheuvals (1981) and, for a recent example, Velásquez-Giraldo et al (2018).

To mitigate the danger of misspecification in fitting the marginal distributions, many empirical copula studies utilize rank-transformed counterparts to the copula data, sometimes referred to as "pseudo data". For each macroeconomic variable, indexed by s, we define the pseudo data as $V_{s,t} = R_{s,t}/(T+1)$, where for each variable, R denotes the rank of the t^{th} observation relative to its own history, for $t = 1, \ldots, T$. The $T+1$ denominator avoids boundary issues.

In our applied work, we consider a similar model space to ABG (2019a) with three macroeconomic variables, so that $S = 3$. Namely, output growth and lagged financial conditions, with lagged output growth also included as a predetermined conditioning variable. We fit the copula density non-parametrically to the three (rank-transformed) pseudo variables to capture dependence. With the dependence estimates conditional on the fitted marginals, this limited information estimation strategy is sometimes known as "inference for margins"; see, Joe (2006).

We fit the multivariate pseudo time series non-parametrically using a kernel density estimator (KDE). Among others, Silverman (1986), Scott (1992), DiNardo and Tobias (2001), Li and Racine (2006) and Adrian, Boyarchenko and Giannone (2019b) adopt KDE methods but not within a copula framework. Working on the S-dimensional unit cube, we utilise a Beta kernel to fit the multivariate distribution, reducing scope for boundary bias; see the discussions in Chen (1999), Charpentier, Fermanian and Scaillet (2007) and Karra (2018). The fitted copula density is:

$$\frac{1}{T}\sum_{t=1}^{T}\prod_{s=1}^{3} K(V_{s,t}, \alpha, \beta) \tag{3}$$

where K denotes the Beta density with parameters $\alpha = \frac{v}{h} + 1$ and $\beta = \frac{1-v}{h} + 1$ and h denotes bandwidth. Following Adrian, Boyarchenko and Giannone (2019b), we select the bandwidth for our kernel density estimator based on out of sample performance for the modern era. We utilize the same bandwidth value for the historical data prior to 1970. Arguably there is greater imprecision in historical National Accounts; see, for example, Romer (1989). We describe our process in detail in the not for publication appendix.

Mindful of the complexity of economic growth in financial crises previously documented by economic historians, the capacity of our empirical copula methodology to handle both non-Gaussianity and non-linearity is appealing. Nevertheless, there is no reason to rule out either Gaussianity or non-linearity a priori. Hence we compare the fit of empirical copulas with both linear and non-linear dependence. And also, for each dependence structure, we explore copula models with Gaussian marginals.

In summary, we fit four specifications in total. First, a benchmark model with Gaussian marginals and linear dependence—equivalent to a conventional linear-Gaussian regression. Second, maintaining the assumption of linearity for dependence, we consider non-Gaussian marginals. Third, we drop the assumption of linear dependence, but adopt Gaussian marginals. And for the final specification, the most general model, we consider non-Gaussian marginals with non-linear dependence.

2.3 Prediction and Evaluation

Prediction involves simulation directly from the copula density on the unit cube for all four specifications. Then, we exploit the appropriate fitted marginal for each specification to produce predictions on the observable scale for output growth. The last step uses the inverse marginal for output growth, $F_1^{-1}(\cdot)$, to generate predictions with the required distribution for the target variable, output growth. We deploy the inverse ECDF for the two non-Gaussian specifications and the inverse Normal for the two Gaussian specifications.[1]

Turning to the assessment of predictive accuracy for our four specifications, we examine both relative Root Mean Squared Errors (RMSE) and a relative entropy based measure of probabilistic accuracy. The RMSE approach computes the square root of the average of squared errors from the conditional mean prediction for each specification, with the lowest RMSE preferred. The idea behind the relative entropy approach is to select a specification which on average gives highest probability to the data. A Bayesian interpretation of this Kullback-Leibler Information Criterion (KLIC) approach to model selection is given by Fernandez-Villaverde and Rubio-Ramirez (2004). See, for further details, the discussions in Kullback and Leibler (1951), Kullback (1987), and Roberston, Tallman and Whiteman (2005). As noted by, for example, Kascha and Ravazzolo (2010), (under some regularity conditions) KLIC optimization is equivalent to minimizing the average logarithmic score (log score) of the densities using the sample data.

3 Data

Given the emphasis in the economic history literature on looking at evidence across a variety of sub-samples and measures of macroeconomic variables, we describe the data considerations in detail. To facilitate comparisons with ABG (2019a), we examine their preferred measures of financial conditions for modern data and, in addition, we explore other measures of financial conditions germane for our historical sub-samples. We also consider similar measures of economic growth to ABG (2019a), albeit using GNP rather than GDP to give a consistent series across historical sub-samples.

Reflecting the heterogeneity in the scale of economic fluctuations through our 100+ year

[1] Therefore, for the specification with linear dependence and non-Gaussian marginals, the implied correlation between observables exhibits (slight) non-linearity as a result of the non-linear transform associated with the inverse ECDF.

span of data, and following much of the historical literature, we divide our sample into four sub-samples and estimate our empirical models for each era. The first of these, 1875:1 to 1913:4, covers the classical Gold Standard. The second, 1919:1 to 1941:3, begins just after World War I and ends just prior to the U.S. entry into World War II.[2] The third era, 1946:1 to 1971:2, corresponds to the Bretton Woods exchange rate system. The fourth era, 1971:3 to 2017:4, starts with the demise of the Bretton Woods system and includes both the Great Moderation and the 2008 financial crisis. Bordo and Haubrich (2010), Jordà et al (2013, 2016) and Schularick and Taylor (2012) use similar era dates. ABG (2019a) consider the most recent (modern) era only.

We follow ABG (2019a) in reporting results for both annualized one and four quarter growth rates of economic growth. We collect our quarterly GNP data from 1875 to 2017 using the FRED database and Balke and Gordon (1986). The length of the sub-samples varies by era, with 154, 90, 101 and 185 observations used for estimation with the one quarter measure, and 148, 87, 98 and 182 used with the four quarter measure. For our first three eras we use the real GNP series from Balke and Gordon to construct the growth measures, and for our last era we use the real GNP series from the FRED database. Further details of the sources and construction of all variables are available in the appendix.

Since the Chicago Fed's National Financial Conditions Index (NCFI) only covers the modern sub-sample from the early 1970s, we explore alternative measures of financial conditions for all eras. These are the credit spread, the term spread and a measure of stock market volatility. For the first era, we use Railway Bond yields from the NBER Macrohistory database, reported by Macaulay (1938), to construct a credit spread. For the remaining three eras, we use the Baa-Aaa spread calculated from FRED data. For term spreads in our first three eras, we follow Balke and Gordon (1986) by using Railway Bond Yields and the Moody's Baa rated bonds to

[2] We find very similar results when our second era spans 1914:1 to 1945:4 and so includes both world wars. These results are reported in the not for publication appendix.

construct a long-term corporate bond yield, and subtract from that the 60-90 day New York City Commercial Paper rate. These data are taken from FRED and Macaulay (1938). For our final era, we use the difference between the 10-Year Treasury Constant Maturity Rate and the 3-Month Treasury Bill rate to represent the term spread, with both series from FRED. Finally, we use Shiller's S&P Composite Index to construct a measure stock market volatility for 1875 to 2017.[3]

Table 1 provides summary statistics for both the one quarter (upper panel) and four quarter measure (lower panel) of output growth by era. The first two rows (in each panel) refer to the mean and standard deviation of the sub-samples, respectively. These illustrate the differences in scale across eras, regardless of the measure of output growth, with variation in the unconditional means and standard deviations. The first era has the highest unconditional mean at 4.08 and the second era has the highest standard deviation at around 12.5. The second era has the lowest unconditional mean at 2.75. This is similar to the mean for the modern sub-sample, 2.77. The modern era has the lowest standard deviation at 3.22. Therefore, while the sub-sample including the Great Moderation and the Great Recession has seen relative stability in economic growth, mean growth is close to that of the sub-sample that includes the Great Depression.[4]

The remaining rows display summary statistics for skewness, kurtosis, and the $p-$values from the Shaprio-Wilk test for non-Gaussianity. All eras exhibit negative skew based on the one quarter measure of growth. Three of the four eras have excess kurtosis, indicating heavy tails in the sub-samples, the exception being the second era which has kurtosis close to three—the value for a Gaussian distribution. The Shapiro-Wilk test indicates significant deviations from Gaussianity for all sub-samples, although for the second era the $p-$value is 13%.

Turning to the summary statistics for the four quarter measure of economic growth dis-

[3]Post-2000 data from Shiller (2000) can be downloaded from http://www.econ.yale.edu/ shiller/data.htm. Further data details are available in the not for publication appendix.

[4]Romer (1989) notes differences in construction methods for GNP across eras.

played in the lower panel of Table 1, the general patterns are similar to those with the one quarter measure, but with lower figures for the standard deviation and kurtosis for each era. Negative skew predominates with this measure, and the $p-$values for the Shapiro-Wilk test are generally below 5 percent. The one exception is the four quarter economic growth measure for the third era, Bretton Woods, which has very slightly positive skew.

The summary statistics for our various measures of financial conditions are reported in Table 2. In general, the financial conditions exhibit positive skew, with only exception being the term spread during the first and last sub-samples with negative skew. There is a general pattern of excess kurtosis across these measures of financial conditions, with the one exception being the term spread. Finally, except for the term spread in the first era, the $p-$values for the Shaprio-Wilk test are below 10 percent and in most cases they are below 1 percent, implying a rejection of the null hypothesis that the data are Gaussian distributed.

Overall, looking across measures of output growth and financial conditions, and sub-samples, there is evidence of non-Gaussianity for both variables—motivating our consideration of non-Gaussian marginal distributions.

4 Results

In this section, we analyse the evidence supporting both non-Gaussianity and non-linearity, and assess whether financial conditions have predictive content for economic growth in our various historical sub-samples. Then, we present our conditional output growth predictive densities and consider the probabilities of output growth events, with an emphasis on well-known historical financial crises. We begin with an ocular assessment of the fitted marginal distributions by era. This provides further support for non-Gaussianity in general but highlights the complex and varied forms of departures from Gaussianity across eras, particularly for measures of financial conditions.

4.1 Fitted Marginal Distributions

Figure 1 provides eight plots displaying the fitted marginal distributions (plotted as probability density functions, smoothed for illustration) from the ECDFs for output growth for each era.[5] We provide results for various economic growth measures, with plots based on the one quarter GNP growth measure in the left column, and the four quarter measure in the right column. Each row of plots refers to a specific sub-sample.

Looking at the top left panel, namely one quarter growth during the Gold Standard era, we see the density exhibits an asymmetric distribution, with negative skew overall, a slightly longer and thinner left tail, and a slightly shorter and fatter right tail. Turning to the modern sub-sample, post-1970, we see a similar pattern in the bottom left panel. ABG (2019a) noted the considerable probability mass to the right of the mode, reflecting the long upswing of the business cycle. Looking at the third row, Bretton Woods, we again see similar features. However, the second era, which includes the Great Depression, has a distinct almost triangular shape indicating considerable probability mass to the left of the mode. Put differently, U.S. output growth was unusually "vulnerable" during the years between the wars, which includes the Great Depression. In general, though, the shapes of the densities across eras (rows) and output growth measures (left and right columns) are similar (ignoring the obvious differences in scale).

Moving onto the marginals for financial conditions, Figure 2 provides the corresponding densities for a variety of measures. The four rows of plots refer to the respective eras, ordered by start date, with (for example) the modern era shown in the bottom row. The three columns from left to right refer to the term spread, the credit spread and S&P volatility, respectively. Looking across eras and measures, all plots display features that are hard to reconcile with Gaussian distributions. Nevertheless, there is heterogeneity across measures for each era. For

[5] A locally adaptive kernel estimation method, developed by Shimazaki and Shinomoto (2010), was used for all results described below.

example, for the first row, the Gold Standard era, the credit spread and S&P volatility both have a long thin right tail, but the term spread has a relatively short (but fat) right tail, and the left tail is longer and thin. Recall that the overall skew for the term spread measure in this era is slightly negative. Although the term spread and S&P volatility are unimodal, the credit spread has a second (local) peak at around 0.5. Generally the credit spread and S&P volatility have similar shapes for each era, with the term spread more idiosyncratic from an ocular perspective.

Overall, we conclude from our ocular assessment that the fitted marginal distrbibutions are typically non-Gaussian but in complex ways that are not easy to reconcile with parametric distributions, especially for measures of financial conditions.

4.2 Assessing Predictive Content

Tables 3 and 4 summarize the in-sample fit of our various specifications using RMSE and the KLIC-based measure of predictive performance, respectively. Although we focus in particular on the additional predictive content from (lagged) financial conditions, recall that throughout we condition on lagged output growth—matching the variables of interest in ABG (2019a).

Each of the 4 panels in both tables corresponds to a single specification. The first panel considers the baseline specification with Gaussian marginals and linear dependence—equivalent to a conventional linear-Gaussian regression. The second panel refers to the specification with non-Gaussian marginals and linear dependence. The third panel covers the case with Gaussian marginals and non-linear dependence. And, the fourth panel refers to the most general specification, with non-Gaussian marginals and non-linear dependence. Rows within each panel consider the four measures of financial conditions below a variant without financial conditions. The eight columns are based on specific sub-samples, with separate results for each measure of output growth.

Looking at Table 3, reporting the relative RMSE of our various specifications, we see from the first panel that adding financial conditions adds little to the fit of the benchmark linear-Gaussian specification. Generally, the gains from including this variable are less than two percent, regardless of the financial conditions measure, and regardless of the measure of output growth.

One exception is the measure of the financial conditions emphasized by ABG (2019a), the Chicago Fed NFCI, reported in the bottom row of the first panel, where the gains are around five percent of RMSE relative to the benchmark for the modern era.[6] A second exception is the term spread, for the first sub-sample for both output growth measures.

Turning to the second panel of Table 3, we see fairly small gains in predicability from adopting non-Gaussian marginals with linear dependence. Without financial conditions, RMSE is close to the linear-Gaussian benchmark. And, adding financial conditions rarely makes much difference to predictability over the corresponding cases in the previous panel or the benchmark. For example, considering the Chicago Fed NFCI, the gains are again close to five percent relative to the benchmark for both measures of output growth in the modern era.

In contrast, for the third panel of Table 3, we see much larger predictability gains. With Gaussian marginals and non-linear dependence ignoring financial conditions, the improvement in RMSE is under 10 percent, regardless of sub-sample or measure of output growth. However, there are considerable gains from including measures of financial conditions for all measures and eras. For example the predictive gain from including the term spread is around 20 percent for the Gold Standard era for the one quarter economic growth measure (relative to the benchmark). For the subsequent eras, the corresponding gains are up to 30 percent for the same measure of output growth, depending on the measure of financial conditions. For the four quarter measure, the gains are in the region of 20 to 40 percent with some variation

[6] Recall that this measure of financial conditions was introduced after 2008 and that data only exist for the modern era.

across eras. Considering the Chicago Fed NFCI, the gain is approximately 15 to 30 percent for both measures of output growth in the modern era. Overall, we conclude that there is strong evidence for the inclusion of financial conditions in all sub-samples, with non-linear dependence and Gaussian marginals. And weak evidence that non-linearity matters even if we do not consider financial conditions.

Considering the fourth panel of Table 3, we see stronger evidence for non-Gaussianity in conjunction with non-linear dependence. Even the case without financial conditions is consistently better than the Gaussian-linear benchmark. Across all sub-samples and for all measures of output growth, RMSEs are below one, in the first row of the fourth panel, with gains in the region of one to 11 percent. Turning to the issue of whether financial conditions matter for this more general specification, we see large predictive gains for all measures and eras in the fourth panel of Table 3. The predictive gain from including, for example, the credit spread is around 20 percent for the Gold Standard era and the one quarter economic growth measure. For the subsequent eras, the gains are as much as 40 percent for the same measures of output growth and financial conditions; see, for example, the second era (including the Great Depression) where the gain is around 42 percent. Similar gains in predictability from including the financial variables are displayed across all sub-samples for the four quarter measure of output growth. For the Chicago Fed NFCI variable, the gain in predictability is in the region of 25 to 30 percent relative to the benchmark.

Overall, we have three findings based on the results presented in Table 3. First, in terms of ranking the predictability of our various specifications, the non-Gaussian and non-linear model is best, followed by the Gaussian and non-linear specification. Second, there is strong evidence for the inclusion of financial conditions in all sub-samples in our preferred specification. Third, even in the specification with Gaussian marginals and non-linear dependence, including financial conditions matters for predictability. So much so, that without financial conditions there is only a modest performance gain from moving beyond the Gaussan-linear benchmark. These findings are robust across measures of output growth and measure of financial conditions.

Turing now to the consideration of the KLIC in Table 4, we see very similar results in terms of probabilistic predictability which are more relevant than RMSE when the concern is the risk to economic growth. Regardless of the measure of output growth, including financial conditions (by any measure) improves predictability in our most preferred non-Gaussian and non-linear specification. As with RMSE, the KLIC results indicate that Gaussian specifications with non-linear dependence rank second overall in terms of predictability if financial conditions are included. From a probabilistic perspective, there is a small predictability gain from adopting the Gaussian and non-linear specification over the benchmark in the absence of financial conditions.

4.3 Conditional Output Growth Densities

Figures 3a through 3d plot the means of the conditional distribution for output growth along with 5^{th} and 95^{th} percentiles over time, along with the data realizations. In all cases the conditional distributions use our preferred empirical specification with non-Gaussian (ECDF) marginals and non-linear dependence. For the modern era we provide results using the NFCI to measure financial conditions to facilitate comparison with ABG (2019a). For the interwar and Bretton Woods sub-samples we report results based on the credit spread measure of financial conditions. Empirical studies using the credit spread include Bernanke (1983), Coe (2002) and Bordo and Haubrich (2010) using historical data, together with Gertler and Koradi (2015) and Caldara and Herbst (2019) using more modern data. For the Gold Standard since our credit spread measure is based on railway bonds only, we focus on results based on the term spread. See Bordo and Haubrich (2008) for a historical study exploring the relationship between the term spread and output growth.[7]

[7]In terms of overall predictability, our results are robust to the choice of financial conditions measure. See the not for publication appendix for sub-sample plots with various measures of financial conditions.

For each of Figure 3a through 3d, in the top row we condition only on lagged output; and, for the bottom row, we condition on both lagged output growth and lagged financial conditions. The columns on the left contain plots for the one quarter growth measure and those on the right contain plots for the four quarter growth measure.

Two features are immediately striking when looking at these figures. First, with the addition of financial conditions, the conditional mean tracks the data much more closely, consistent with the large drop in RMSE reported in the last panel of Table 3. This characteristic emerges for realizations of output growth in the lower tail of the unconditional distribution, and also for realizations closer to the historical mean. Therefore, while our discussion below focuses on episodes considered as financial crises, our results suggest that financial conditions also contain information about movements in output growth near the central probability mass. Second, the spread between the 5^{th} and 95^{th} percentiles is much smaller in the lower panels—indicating reduced uncertainty when accounting for financial conditions. These features occur regardless of the measures of output growth.

Looking more closely at Figure 3d, which refers to the modern era, the conditional mean declines around 2008 for both measures of output growth when including financial conditions in the bottom row. Furthermore, there is a coincident drop in the 5^{th} percentile—consistent with the conclusion that financial conditions contributed to increased growth vulnerabilities.

The Jordá, Schularick and Taylor (2017) Macrohistory Database identifies three crisis, in 1893, 1907 and 1929, that of particular interest to economic historians. Figures 3a and 3b display features for these crises resembling those for the 2008 crisis (in Figure 3d). Namely, coincident declines in the conditional mean and 5^{th} percentile for both one quarter and four quarters measures of economic growth. We explore the risks to economic growth in further detail in the next sub-section.

4.4 Probabilities of Low and High Output Growth Events

ABG (2019a) report that deteriorating financial conditions accompany increased risk of low economic growth events and that the probability of high growth events exhibit stability.

Figures 4a through 4d plot the probabilities of high and low growth events over time for our preferred specification with non-Gaussian (ECDF) marginals and non-linear dependence, conditioning on financial conditions (and lagged output growth).[8] The choice of the high growth event and the low growth event is somewhat arbitrary in this approach. When looking at growth rates in the lower tail, Jordá et al (2017) use the bottom decile. We define a low growth event as output growth below the 10^{th} percentile, and we define a high growth event as above the 90^{th} percentile, for each era.[9] Figures 4a through 4d plot probabilities without financial conditions in the top row and provide probabilities with financial conditions in the bottom row. The plots in the left column indicate the probabilities of high and low growth in the next quarter, while the plots in the right column indicate the corresponding probabilities for the next year. In each panel, the probabilities of low growth are depicted by red bars, measured upwards from zero. The probabilities of high growth are depicted by blue bars, measured downwards from one.

Comparing the top and the bottom row for Figures 4a through 4d, respectively, financial conditions result in increased variation in the conditional probabilities, regardless of the measure of output growth. Looking more closely at the interwar era for example, the probabilities of the low growth event are typically in the range of 0.1 to 0.2, and only above 0.2 on a few occasions absent financial conditions. In particular, there are only modest increases in the conditional probability (to around 0.35) following the 1929 stock market crash and during the Great Depression. However, when we include financial conditions, these probabilities peak

[8] We use the same measures as in Figure 3. Corresponding plots for all measures of financial conditions are reported in the not for publication appendix.

[9] We supply corresponding plots for alternative definitions of high and low growth in the not for publication appendix.

much higher following the 1929 crash and remain above 0.5 for much of the Great Depression.

Increased risk for low growth events also occurs during other financial crises when accounting for financial conditions. In the first era, for example, there are large increases in the probability of low growth events for both the 1893 and 1907 crises displayed in Figure 4a. Figure 4d displays the higher probability of low growth during the Global Financial Crisis.

Recall at our empirical copula approach allows for non-Gaussianity in the conditional densities in both tails—giving the flexibility to track upside and downside risk, simultaneously. ABG (2019a) report that downside risks are related to financial conditions, but that upside risks are not. In contrast, our evidence suggests a strong influence of financial conditions on both upside and downside risks to economic growth. Financial conditions sharpen the predictive densities for economic growth in each era, represented by Figures 4a through 4d.

4.5 Selected Financial Crises

Figure 5 plots the conditional predictive densities for output growth during four financial crises. The panels on the left refer to output growth in the quarter following the financial crisis and the panels on the right refer to output growth in the year following the financial crisis. In each case these are predictive densities for future output growth conditional on current financial conditions and current output growth. Each row refers to a specific crisis. These are the second quarter of 1893, the fourth quarter of 1907, the fourth quarter of 1929 and the fourth quarter of 2008. In each panel, we plot the predictive densities with and without (lagged) financial conditions, for our preferred non-Gaussian and non-linear specification (conditioning on lagged output growth throughout).

The top row of Figure 5 plots the predictive densities for the 1893 crisis, associated with a fall in confidence regarding the Gold Standard. Arguably, the Sherman Silver Purchase Act of 1890 and the decline in Treasury gold reserves after December 1892 unsettled financial markets;

see, for example, the discussion in Rockoff (1990). An alternative view, provided by Friedman and Schwartz (1963), argues that concerns over bank solvency led to bank runs. We assume that the 1893 crisis occurred in the second quarter of 1893 to coincide with the decline in stock prices and the bank runs in June of that year. The top row of Figure 5 shows the predictive densities for output growth in both the subsequent quarter and year.[10] For the one quarter measure of economic growth (left column), the conditional predictive density without financial conditions (in black) displays evidence of negative skew, with a mode slightly below zero. he corresponding predictive density with financial conditions (in red) exhibits bi-modality, with twin peaks in probability at approximately zero and -25 percent economic growth. For the year after the 1893 crisis, the twin peaks occur at approximately -10 and two percent economic growth. Disregarding financial conditions for the one year ahead case results in just a single probability peak at around seven percent economic growth. Financial conditions radically change the assessments of downside risks to economic growth around the 1893 crisis.

The second row of Figure 5 plots the predictive densities for the 1907 crisis, which arguably originated with runs in the "shadow banking" sector; see, for example, Frydman et al (2015). According to Friedman and Schwartz (1963), the downturn began in May 1907 and was amplified by the failure of the Knickerbocker Trust in October, leading to a run on New York trust companies. Outside the New York Clearing House system, these companies had no access to the funds organized by J.P. Morgan, in part financed by the Treasury, that contributed to halting the crisis. Frydman et al (2015) draw a parallel between these funds and lending by the Federal Reserve during 2008. We assume that the 1907 crisis occurred in the fourth quarter of 1907. The second row of Figure 5 shows the predictive densities with (red) and without financial conditions (black) for the subsequent quarter (left column) and year (right column). The predictive densities indicate higher probability of negative economic growth with financial conditions for both horizons at around 90 percent (one quarter) and 25

[10] As for Figures 3 and 4, we use the term spread to represent financial conditions in this row.

percent (four quarter). As with the 1983 crisis, there is evidence of bi-modality only when accounting for financial conditions.

The third row of Figure 5 plots the predictive densities for output growth following the 1929:4 stock market crash. At both the one quarter and year ahead horizons, the probability of negative output growth is markedly greater conditional on financial conditions. For the one quarter ahead (left column), the probability of negative growth is nearly 100 percent. For the year ahead (right column) the corresponding probability is around 80 percent. Without financial conditions, the probabilities drop to around 70 percent and 30 percent, respectively. Interestingly, although the four quarter ahead predictive density is bi-modal, the one quarter ahead predictive is unimodal when accounting for financial conditions.

The bottom row of Figure 5 plots predictive densities for the fourth quarter of 2008. We use the same NFCI measure of financial conditions as ABG (2019a).[11] The lower conditional predicted mean and the (previously noted) increase in downside risks for both the quarter and the year following 2008:4 are apparent from the predictive densities with financial conditions (red) and without (black). However, the impact of financial conditions on downside risk are more modest for 2008 than for the earlier crises.

We emphasize the bi-modality evident in five out of the six cases when conditioning on financial conditions for the 1893, 1907 and 1929 crises. This property contributes considerably to the downside risk assessments during historical crises. Ignoring financial conditions results in weaker downside risk and typically unimodal probability density functions. Although financial conditions also influence the assessment of downside risk for the 2008 crisis, the effect is less pronounced and the predictive densities are unimodal. The historical evidence presented here suggests even stronger impacts of financial conditions on downside risk for economic growth than for the most recent crisis.

[11] ABG (2019a) report similar results using a term spread, a credit spread and a measure of stock market volatility. We report qualitatively similar results in the not for publication appendix, along with results using alternative measures of financial conditions for other eras.

5 Conclusion

Our probabilistic analysis of the historical evidence supports the empirical connection between financial conditions and economic growth in the United States, building on earlier work by Bordo and Haubrich (2010), Schularick and Taylor (2012), Jordà, Schularick and Taylor (2013, 2016), Jalil (2015), Adrian, Boyarchenko and Giannone (2019a) and others. Our consideration of a variety of measures of economic growth and financial conditions establishes the robustness of the relationship across sub-samples of quarterly historical data from 1875 to 2017.

Our paper differs substantially from those in the extant literature. This study is the first to provide probabilistic assessments of the impact of financial conditions on both the upside and downside risks to economic growth for historical data. We find that both non-Gaussianity and non-linearity shape the historical relationship between these important macroeconomic variables.

Given the effectiveness of the NFCI as a predictor for economic growth on modern data documented by Adrian, Boyarchenko and Giannone (2019a), a corresponding index suitable for historical analysis would be helpful for subsequent research. This would permit probabilistic assessments for historical events based on a still wider set of financial conditions measures.

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Table 1: Summary Statistics – Output Growth

	1875:3-1913:4	1919:2-1941:3	1946:2-1971:2	1971:4-2017:4
Annualized 1-Quarter Growth Rate				
Mean	4.08	2.75	3.31	2.77
Standard Deviation	8.43	12.5	4.47	3.22
Skewness	-0.37	-0.41	-0.51	-0.53
Kurtosis	5.34	2.70	4.61	5.67
Shapiro-Wilk $p-$value	< 0.001	0.133	0.011	< 0.001
Number of Observations	154	90	101	185
	1877:1-1913:4	1920:1-1941:3	1947:1-1971:2	1972:3-2017:4
4-Quarter Growth Rate				
Mean	4.26	2.81	3.51	2.80
Standard Deviation	6.02	9.07	3.12	2.22
Skewness	-0.23	-0.40	0.05	-0.52
Kurtosis	3.59	2.21	3.23	4.00
Shapiro-Wilk $p-$value	0.031	0.009	0.728	< 0.001
Number of Observations	148	87	98	182

Notes: One quarter growth rates are reported at an annualized rate. The Shapiro-Wilk $p-$value refers to a test of the null hypothesis that the data are Gaussian.

Table 2: Summary Statistics – Financial Variables

	1875:1-1913:4	1919:1-1945:3	1946:1-1971:2	1971:3-2017:4
Term Spread				
Mean	-0.27	3.22	1.38	1.72
Standard Deviation	1.23	1.85	0.80	1.27
Skewness	-0.21	0.20	0.38	-0.71
Kurtosis	3.26	2.33	4.05	3.41
Shapiro-Wilk $p-$value	0.424	0.089	0.022	< 0.001
Credit Spread				
Mean	0.81	1.99	0.66	1.09
Standard Deviation	0.32	0.86	0.20	0.46
Skewness	0.56	1.64	1.42	1.82
Kurtosis	2.88	6.89	6.31	7.57
Shapiro-Wilk $p-$value	< 0.001	< 0.001	< 0.001	< 0.001
S&P Volatility				
Mean	2.42	4.20	2.54	2.64
Standard Deviation	1.48	3.60	1.40	1.69
Skewness	1.45	2.71	0.87	1.04
Kurtosis	6.21	14.1	3.92	4.26
Shapiro-Wilk $p-$value	< 0.001	< 0.001	0.001	< 0.001
National Financial Conditions Index				
Mean				0.013
Standard Deviation				0.99
Skewness				1.90
Kurtosis				6.15
Shapiro-Wilk $p-$value				< 0.001
Number of Observations	156	91	102	186

Note: The Shapiro-Wilk $p-$value refers to a test of the null hypothesis that the data are Gaussian.

Table 3: Root Mean Square Errors

	1 Quarter Growth				4 Quarter Growth			
	1875:3-1913:4	1919:2-1941:3	1946:2-1971:2	1971:4-2017:4	1877:1-1913:4	1920:1-1941:3	1947:1-1971:2	1972:3-2017:4
Gaussian Marginals and Linear Dependence								
No Financial Variable	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Term Spread	0.947*	0.998	0.985	0.989	0.884***	0.971	0.937*	0.920***
Credit Spread	0.998	1.002	1.001	0.996	0.985	0.988	0.999	1.001
S&P Volatility	0.994	0.999	0.979	0.988	0.998	1.003*	0.956*	0.996
Chicago Fed NFCI				0.956*				0.953*
Non-Gaussian Marginals and Linear Dependence								
No Financial Variable	0.999	1.003	1.005	1.005	1.002	1.005	1.001	0.993
Term Spread	0.945*	1.002	0.993	0.997	0.887***	0.977	0.955	0.925***
Credit Spread	0.998	1.004	1.009	0.997	0.994	0.991	0.998	0.994
S&P Volatility	0.993	1.002	0.978	0.996	1.000	1.007	0.947	0.989
Chicago Fed NFCI				0.974				0.955**
Gaussian Marginals and Non-linear Dependence								
No Financial Variable	0.982	0.928**	0.942**	0.988	0.963**	0.943*	0.875***	0.939***
Term Spread	0.784***	0.672***	0.758***	0.805***	0.628***	0.673***	0.713***	0.714***
Credit Spread	0.857***	0.669***	0.693***	0.822***	0.708***	0.724***	0.678***	0.808***
S&P Volatility	0.853***	0.698***	0.657***	0.824***	0.805***	0.762***	0.687***	0.807***
Chicago Fed NFCI				0.836***				0.691***
Non-Gaussian Marginals and Non-linear Dependence								
No Financial Variable	0.969**	0.919**	0.955**	0.990	0.954**	0.950**	0.889**	0.936***
Term Spread	0.757***	0.632***	0.668***	0.764***	0.619***	0.623***	0.531***	0.684***
Credit Spread	0.802***	0.584***	0.665***	0.778***	0.621***	0.574***	0.577***	0.742***
S&P Volatility	0.753***	0.549***	0.624***	0.780***	0.733***	0.621***	0.569***	0.783***
Chicago Fed NFCI				0.752***				0.681***

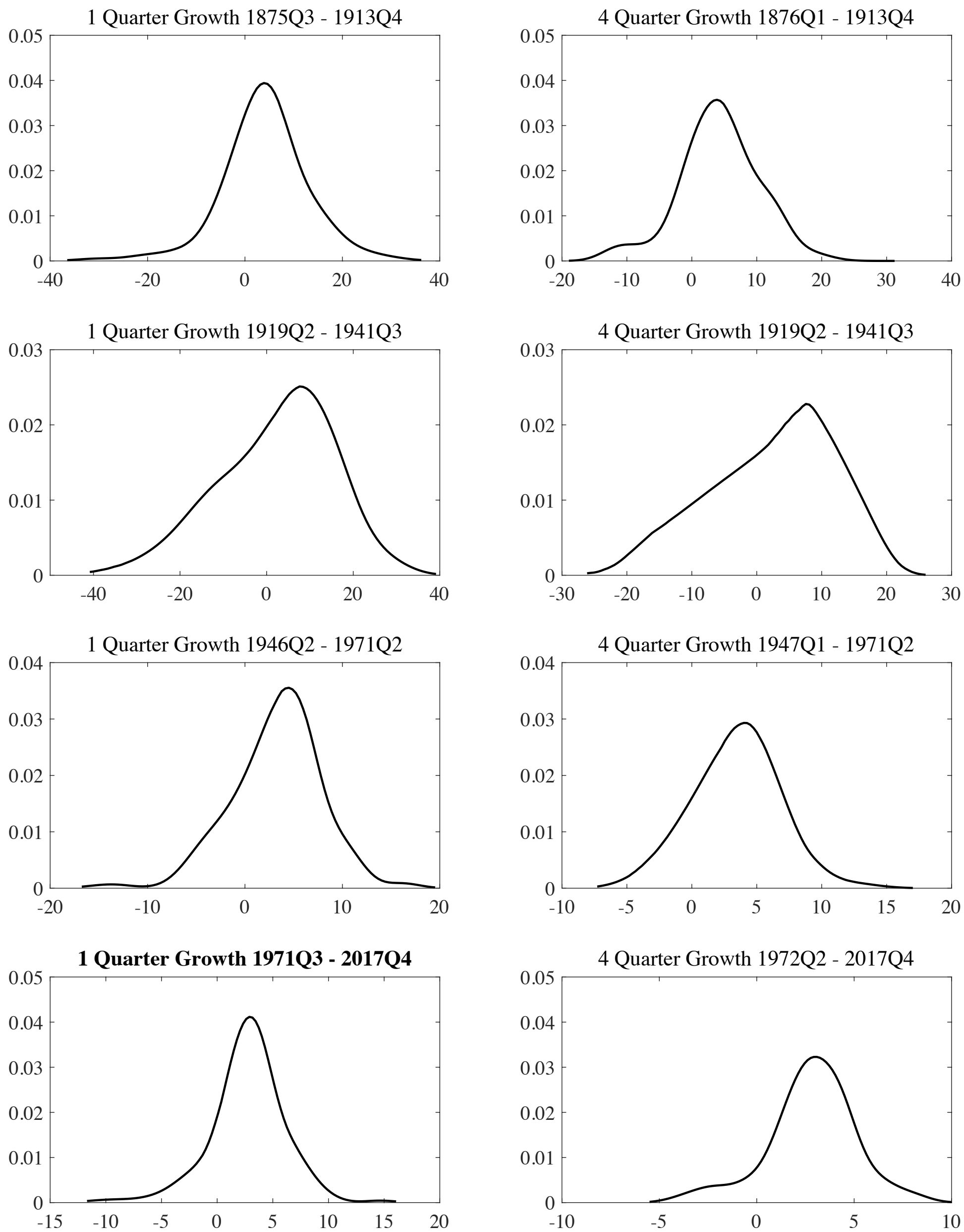
Note: Root Mean Squared Errors are reported relative to the benchmark model with Gaussian marginals, linear dependence and no financial variable. Rejections of the null hypothesis of equal predictability using a Diebold-Mariano (1995) type test with the Harvey, Leybourne and Newbold (1997) correction for autocorrelation and size sample are denoted by $*$ (10%), $**$ (5%) and $***$ (1%).

Table 4: KLIC-Based Measure of Predictive Accuracy

	1 Quarter Growth				4 Quarter Growth			
	1875:3-1913:4	1919:2-1941:3	1946:2-1971:2	1971:4-2017:4	1877:1-1913:4	1920:1-1941:3	1947:1-1971:2	1972:3-2017:4
Gaussian Marginals and Linear Dependence								
No Financial Variable	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Term Spread	0.982**	1.000	0.995	0.996	0.969***	0.993	0.983	0.980**
Credit Spread	0.997*	1.000	0.999	1.000	0.996	0.998	1.000	1.000
S&P Volatility	0.998	1.000	0.993	0.994*	0.999	1.001**	0.989*	1.000
Chicago Fed NFCI				0.987**				0.987*
Non-Gaussian Marginals and Linear Dependence								
No Financial Variable	0.980**	0.995	0.991	0.982**	0.991**	0.993	0.998	0.983***
Term Spread	0.967**	0.995	0.985**	0.980***	0.961***	0.986	0.986	0.968***
Credit Spread	0.979**	0.995	0.990*	0.980*	0.990**	0.990	0.998	0.984***
S&P Volatility	0.979**	0.995	0.982**	0.980**	0.991**	0.994	0.985*	0.983**
Chicago Fed NFCI				0.975***				0.976***
Gaussian Marginals and Non-linear Dependence								
No Financial Variable	0.942***	0.942***	0.945***	0.993	0.943***	0.958***	0.957**	0.951***
Term Spread	0.804***	0.820***	0.812***	0.860***	0.809***	0.832***	0.862***	0.818***
Credit Spread	0.822***	0.830***	0.823***	0.891***	0.814***	0.869***	0.843***	0.848***
S&P Volatility	0.841***	0.845***	0.814***	0.866***	0.863***	0.876***	0.849***	0.840***
Chicago Fed NFCI				0.915***				0.860***
Non-Gaussian Marginals and Non-linear Dependence								
No Financial Variable	0.935***	0.945***	0.949***	0.951***	0.939***	0.953***	0.934***	0.936***
Term Spread	0.794***	0.815***	0.796***	0.798***	0.802***	0.803***	0.781***	0.781***
Credit Spread	0.793***	0.809***	0.798***	0.800***	0.790***	0.809***	0.785***	0.807***
S&P Volatility	0.790***	0.805***	0.791***	0.801***	0.808***	0.812***	0.777***	0.802***
Chicago Fed NFCI				0.800***				0.804***

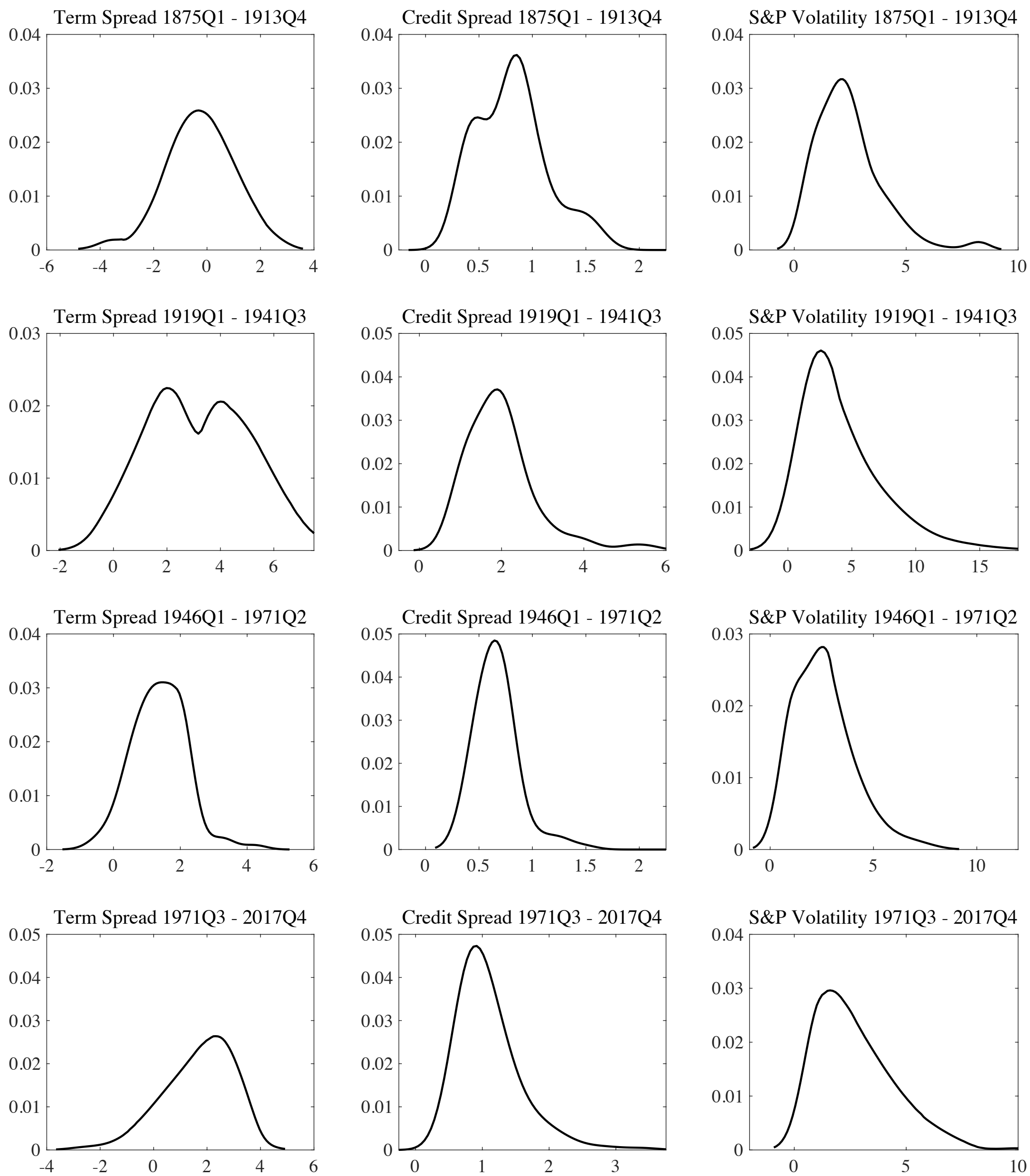
Note: This table displays average log score relative to the benchmark model with Gaussian marginals, linear dependence and no financial variable. Rejections of the null hypothesis of equal predictability using a Diebold-Mariano (1995) type test for the log score are denoted by $*$ (10%), $**$ (5%) and $***$ (1%).

Figure 1: Fitted Marginal Distributions for Output Growth



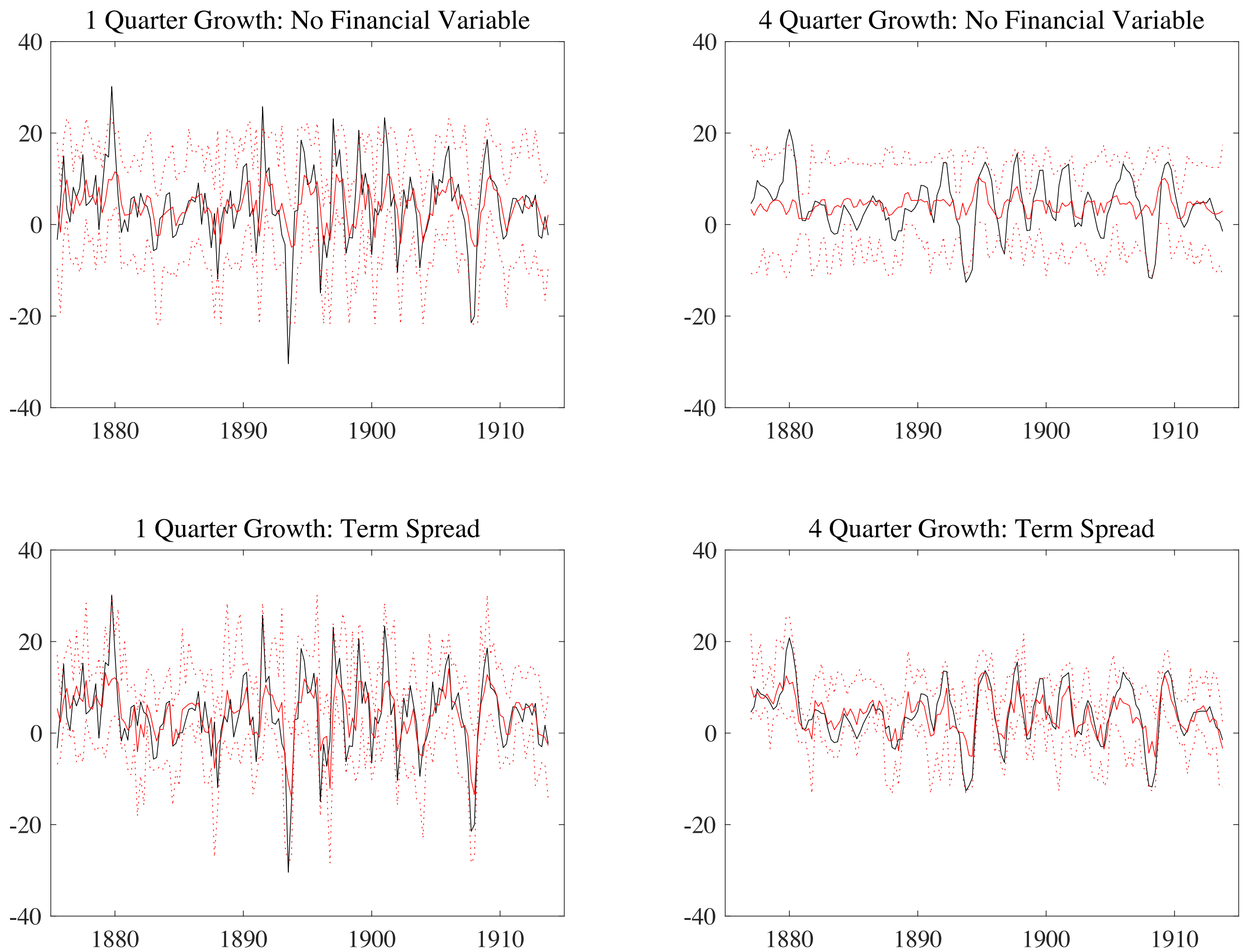
Note: The panels depict the marginal distribution for output growth in each era, fitted using the SSV method of Shimazaki and Shinomoto (2010) and plotted as PDFs. Matlab code available at https://www.mathworks.com/matlabcentral/fileexch ssvkernel-x-tin.

Figure 2: Fitted Marginal Distributions for Financial Variables



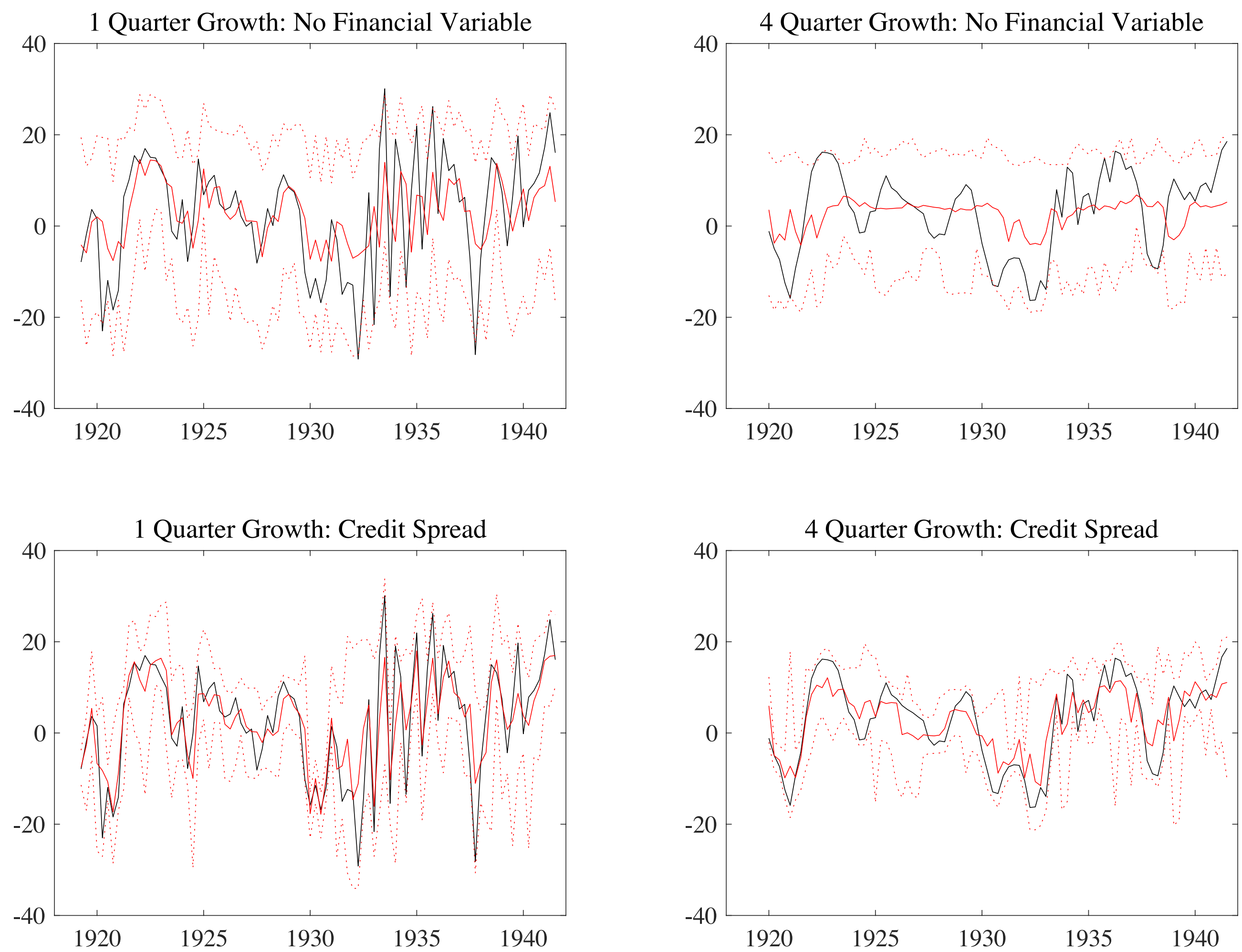
Note: The panels depict the marginal distribution for the term spread, the credit spread and a measure of stock market volatility in each era, fitted using the SSV method of Shimazaki and Shinomoto (2010) and plotted as PDFs. Matlab code available at https://www.mathworks.com/matlabcentral/fileexchange/37374-ssvkernel-x-tin.

Figure 3a: Conditional Densities 1875 - 1913



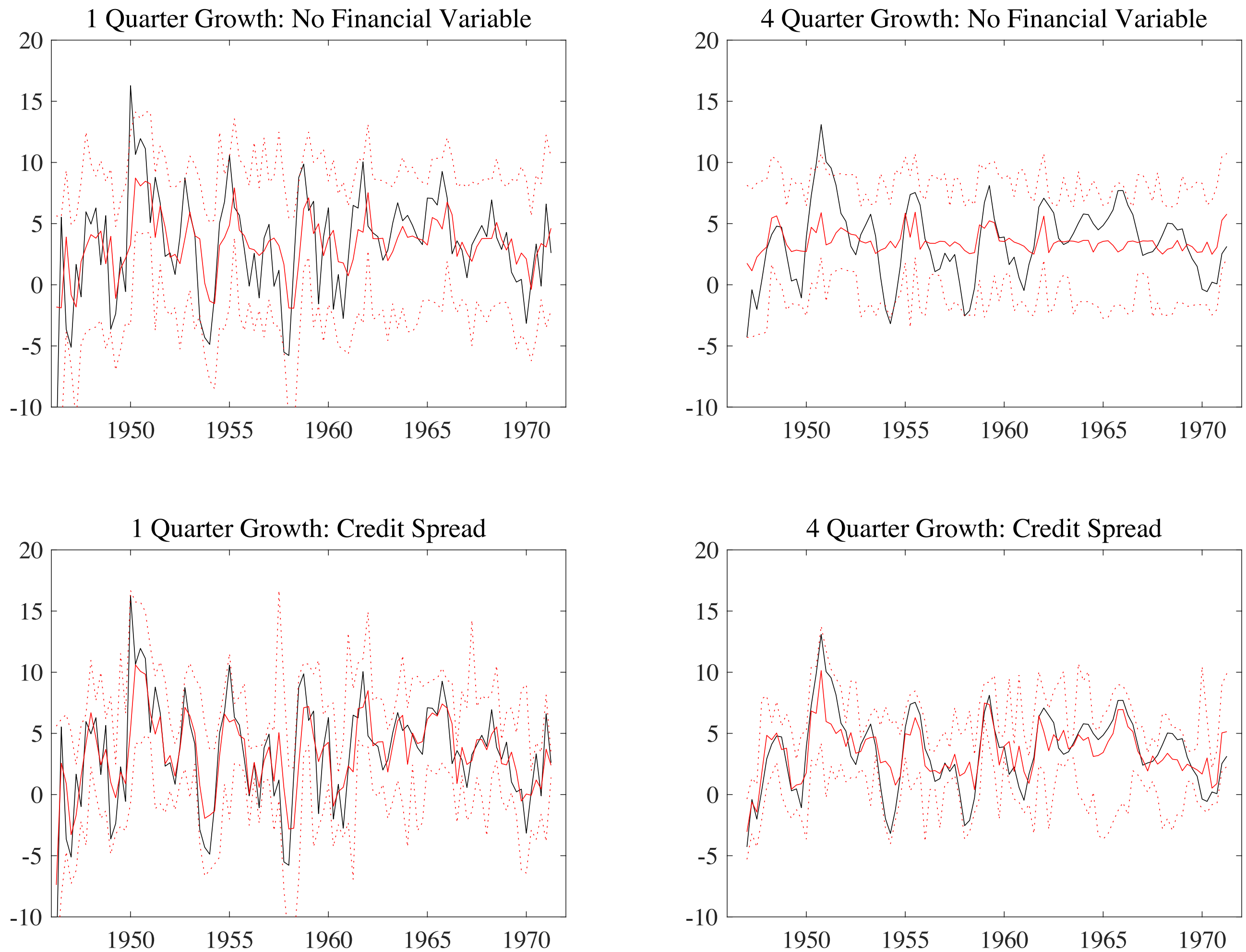
Note: In each panel, the solid line black line depicts the realizations and the solid red line depicts the mean of the conditional density for output growth based on a model with non-Gaussian marginals and non-linear dependence. The dotted red lines depict the 5^{th} and 95^{th} percentiles.

Figure 3b: Conditional Densities 1919 - 1941



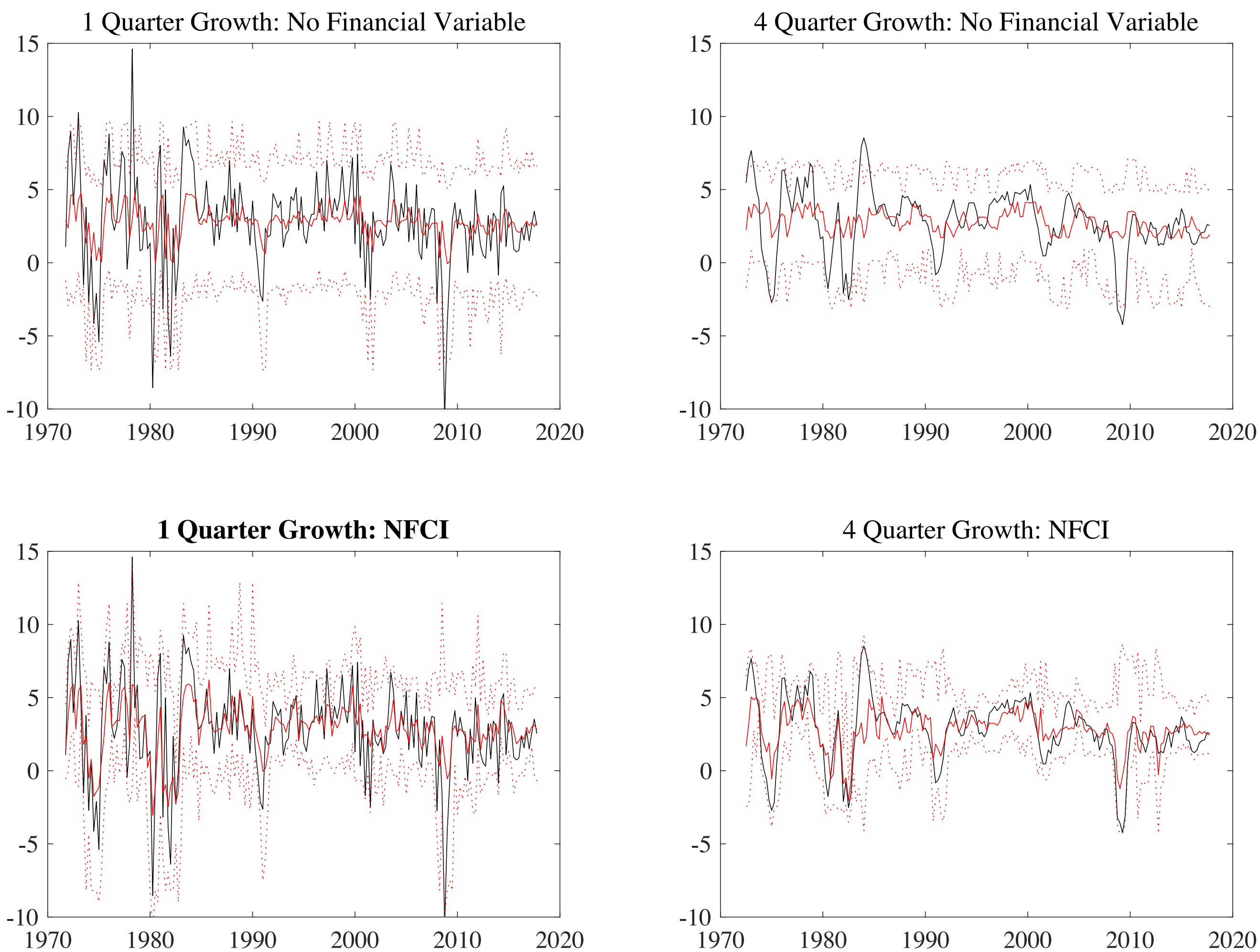
Note: See notes to figure 3a.

Figure 3c: Conditional Densities 1946 - 1971



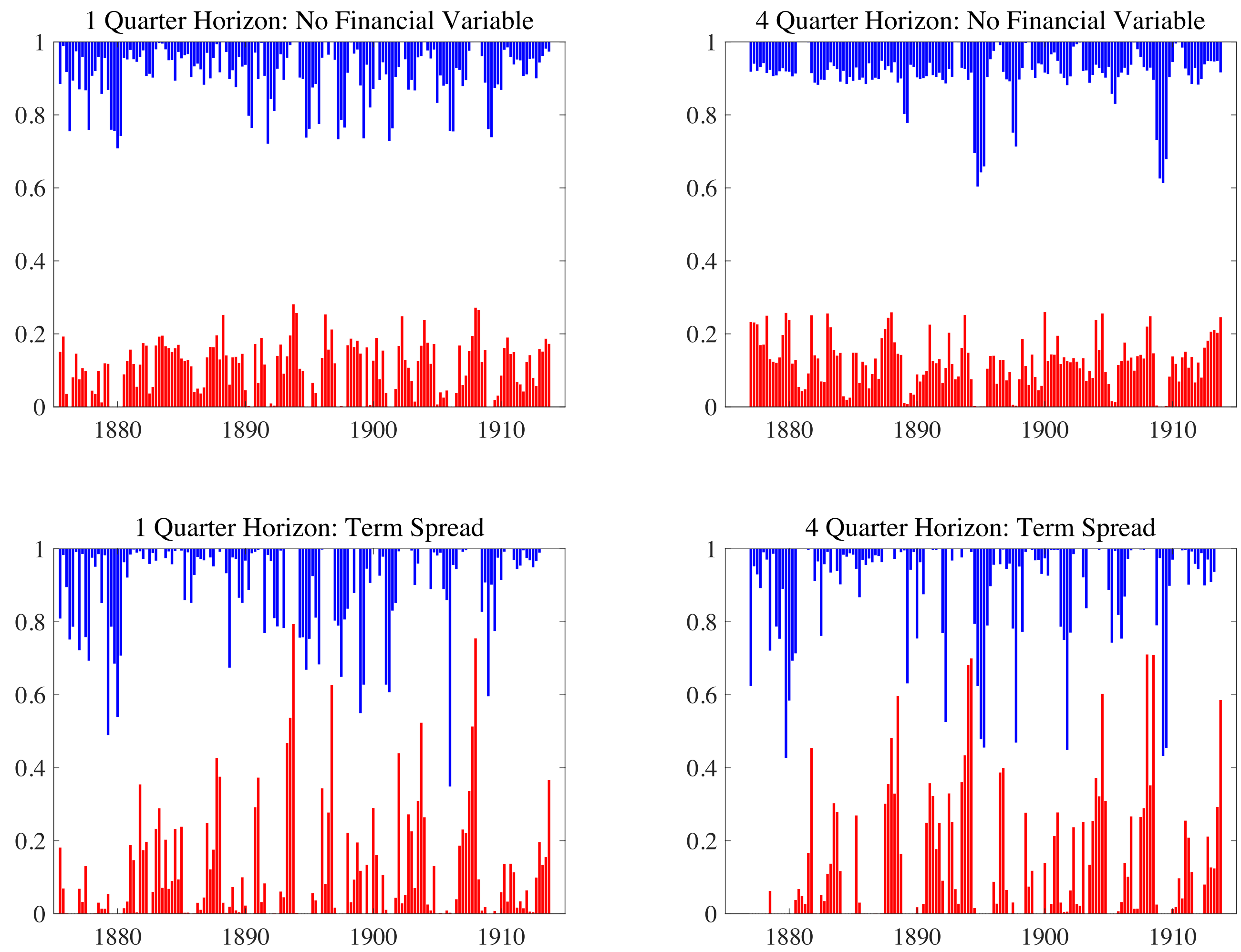
Note: See notes to figure 3a.

Figure 3d: Conditional Densities 1971 - 2017



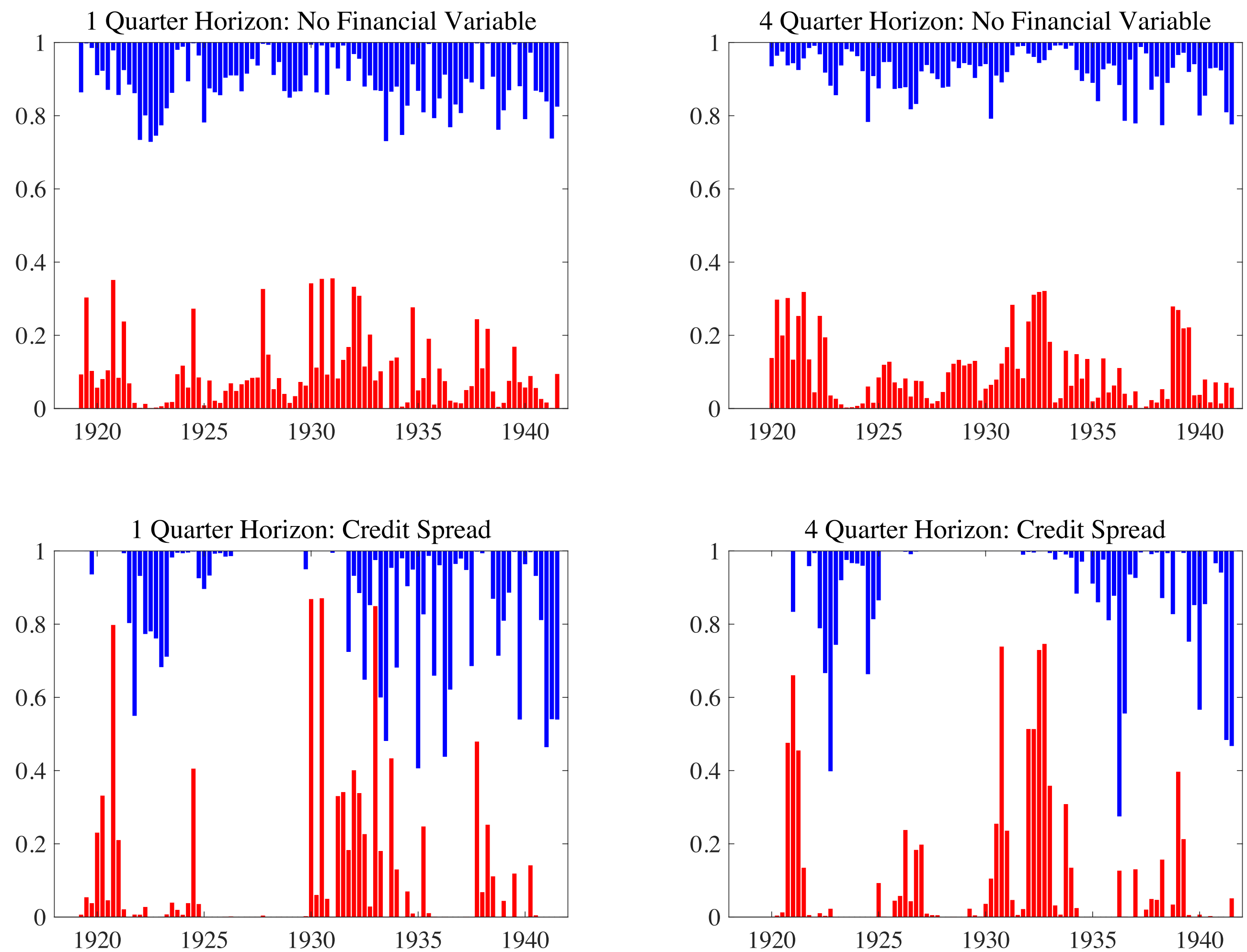
Note: See notes to figure 3a.

Figure 4a: Probability of Tail Events 1875 - 1913



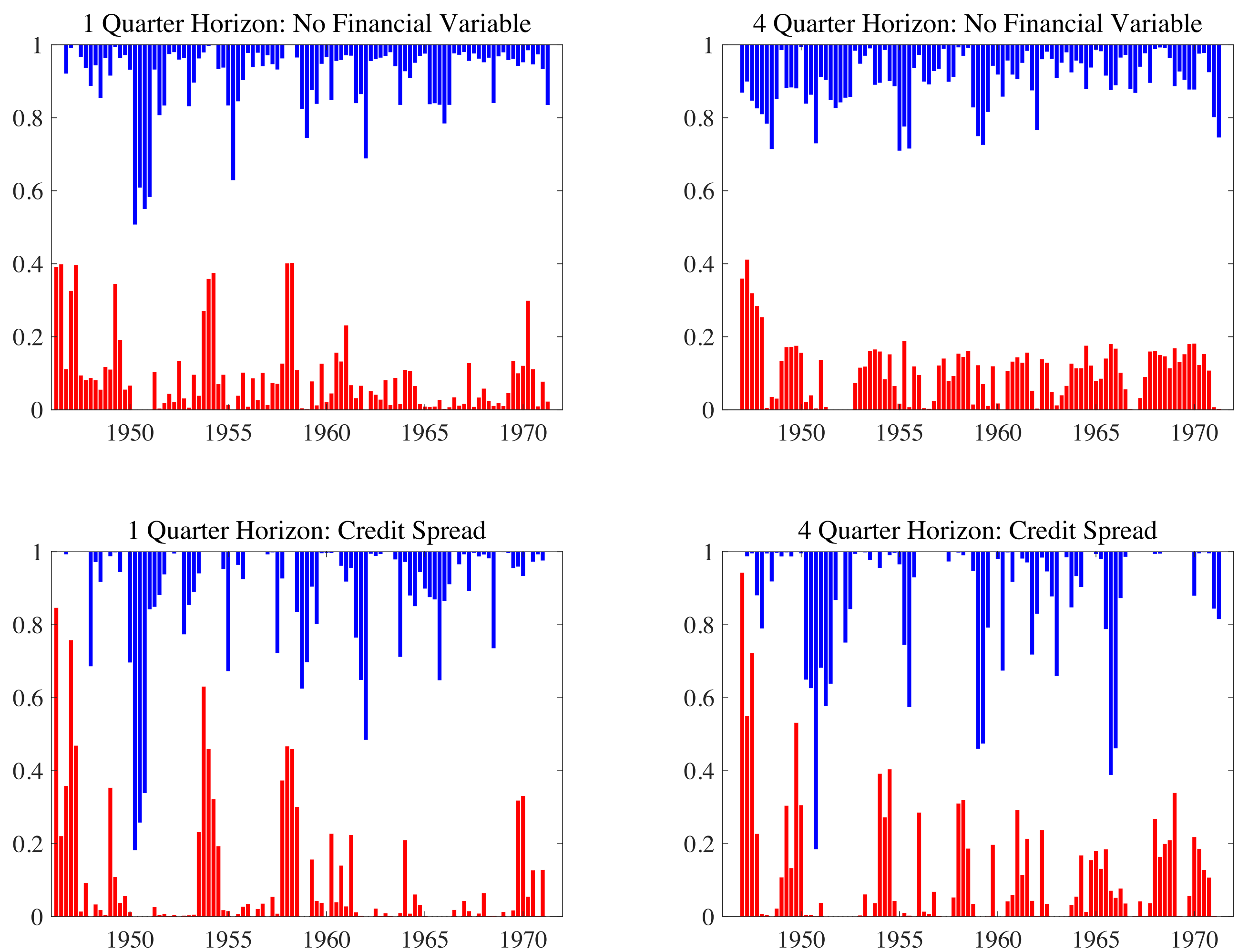
Note: In each panel, the red bars depict the conditional probability assigned to low output growth events below the 10^{th} percentile of the unconditional distribution for the relevant sub-sample and are measured upwards from zero. The blue bars depict the conditional probability of high output growth events greater than the 90^{th} percentile of the unconditional distribution for the relevant sub-sample and are measured downwards from one. All specification use non-Gaussian marginals and non-linear dependence, conditional on either current output growth alone or both current output growth and financial conditions.

Figure 4b: Probability of Tail Events 1919 - 1941



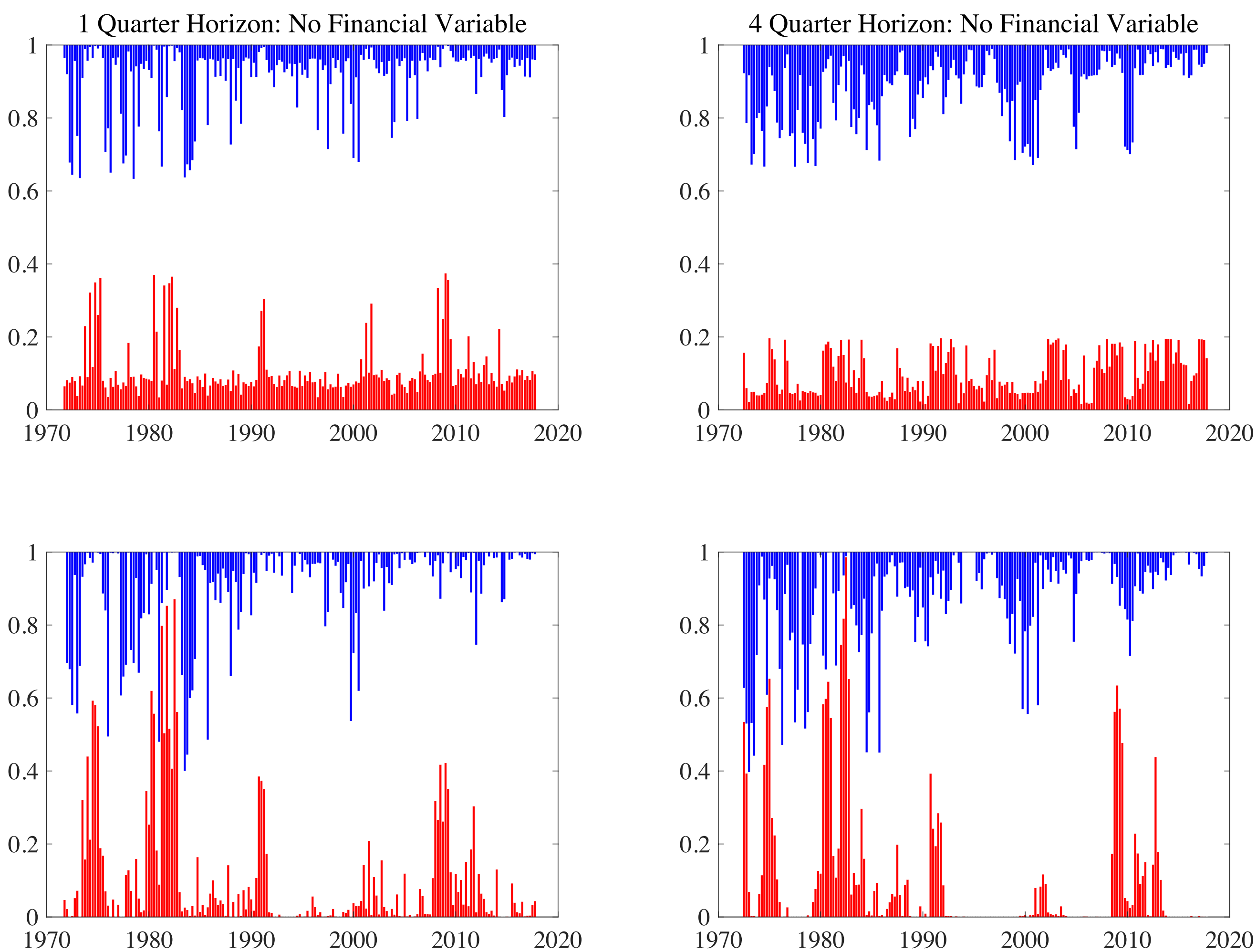
See notes to Figure 4a.

Figure 4c: Probability of Tail Events 1946 - 1971



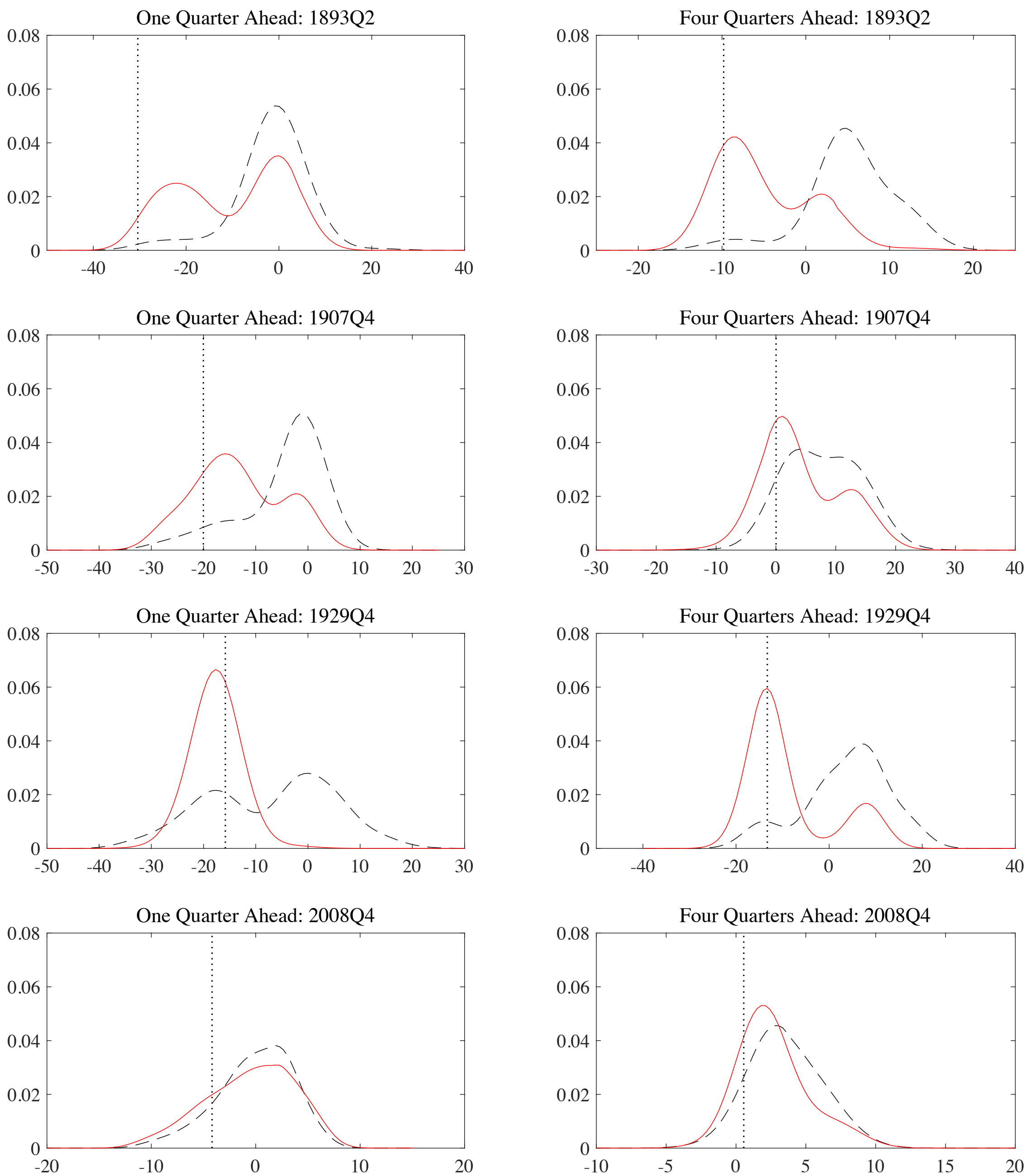
See notes to Figure 4a.

Figure 4d: Probability of Tail Events 1971 - 2017



See notes to Figure 4a.

Figure 5: Predictive Densities For Selected Financial Crises



Note: In each panel the dashed black line depicts the conditional density based on the specification with non-Gaussian marginals and non-linear dependence, without financial conditions. The solid red line depicts the equivalent density accounting for financial conditions. The measures of financial conditions are the term spread (first two rows), the credit spread (third row) and the NFCI (fourth row). The financial crises dates are 1893:2, 1907:4, 1929:4 and 2008:4. The left (right) panels display the densities for output growth in the subsequent quarter (year).