# Reassessing The Dependence Between Economic Growth and Financial Conditions Since 1973*

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**Abstract**

Adrian, Boyarchenko and Giannone (2019, ABG) adapt Quantile Regression (QR) methods to examine the relationship between U.S. economic growth and measures of financial conditions. We confirm their findings using their sample which ends in 2015:4. Mindful of the importance of the Covid-19 pandemic, we update the sample to 2020:4 and find that there is still a negative relationship, albeit with some attenuation. We reassess the empirical evidence exploiting computationally-convenient copula QR methods in a manner that is robust to distributional features. Our copula-based analysis demonstrates the robustness of ABG's characterisation of the dependence.

**Keywords**: Vulnerable Growth, Quantile Regression, Copula Modelling

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# 1 Introduction

Figure 1 plots quarterly data for U.S. economic growth and (lagged) financial conditions, using a one-quarter annualised measure of economic growth and the National Financial Conditions Index (produced by the Chicago Fed). Notable features since the turn of the century include: the negative co-movements during the Global Financial Crisis (GFC); the relative stability of both variables during the post-GFC recovery; the sharp drop in economic growth during the second quarter of 2020 to (approximately) -30 percent (roughly four times the size of the drop in 2008:4); and, the rebound in economic growth in the third quarter of 2020.

Adrian, Boyarchenko and Giannone (2019, ABG) argue that economic growth and financial conditions exhibit a negative relationship utilising Quantile Regression (QR) methods. Based on a sample ending in 2015:4, which hereafter we refer to as "ABG's sample", their analysis does not include the last 20 quarterly observations displayed in Figure 1, and in particular, those associated with the onset of the Covid-19 pandemic during 2020.

In the next two sections of this paper, we outline ABG's approach and then contrast with our computationally-convenient copula methodology. By exploiting the information in the rank data for each variable, we fit dependence in a manner that is robust to distributional features. In the results section, we demonstrate that when considering the extended sample, some attenuation in the key economic relationship is apparent with conventional QR. Nevertheless, our copula methods confirm that the dependence is similar to that in ABG's sample.

# 2 ABG's Data, Model Space and Quantile Regressions

ABG consider the relationship between U.S. economic growth and lagged financial conditions, including lagged economic growth as a conditioning variable. Their analysis is based on a sample from 1973:1 to 2015:4 and encompasses several measures of economic growth and financial conditions. The main results presented in their paper focus on the relationship between one-quarter (and four-quarter) economic growth and the (lagged) National Financial Conditions Index (NFCI).[1] The NFCI is a weekly factor model based index of financial conditions derived from over 100 indicators from money, debt and equity markets. Positive values for the NFCI imply that financial conditions are tighter than average by construction. We convert the weekly data to the quarterly frequency by averaging, following ABG.[2]

ABG's empirical strategy comprises two main elements. First, the estimation of the conditional quantile function using conventional multivariate QR. Second, the analysis of conditional forecast densities for economic growth based on interpolation with the skew-t distribution (Azzalini and Capitanio, 2003). In this paper, we focus on the multivariate QR approach. (We present forecast densities for the extended sample in the not for publication appendix.)

Recent macro-econometric studies using (modified) QR methods include: Giglio, Kelly, and Pruitt (2016), De Nicolo and Lucchetta (2017), Chavleishvili and Manganelli (2019), Ferrara, Mogliani, and Sahuc (2019) and Loria, Matthes and Zhang (2019). The QR approach also underlies the IMF's bilateral surveillance tool known as "growth at risk" (Adrian et al., 2018).[3] Reichlin, Ricco, and Hasenzagl (2020) argue that other indicators than the NFCI give better risk signals for economic growth. Carriero, Clark and Marcellino (2020) argue that stochastic volatility models provide a robust alternative methodology to QR.

Following ABG, denote the target variable, economic growth in $t+1$ as $y_{t+1}$, and the explanatory variable vector, comprising a constant, economic growth and NFCI in $t$, as $x_t$. The QR approach selects parameters to minimise the quantile weighted absolute value of errors. That is,

$$\hat{\beta}_\tau \;=\; \underset{\beta_\tau \in \mathbb{R}^k}{\arg\min} \sum_{t=1}^{T-1} (\tau \cdot \mathbf{I}_{(y_{t+1} \geq x_t\beta)} |y_{t+1} - x_t\beta_\tau| + (1-\tau) \cdot \mathbf{I}_{(y_{t+1} < x_t\beta)} |y_{t+1} - x_t\beta_\tau|),$$

where $I(\cdot)$ denotes the indicator function and $k$ denotes the number of explanatory vari-

[1] Our annualised real GDP growth data are calculated using series GDPC1 from FRED. ABG use series A191RL1Q225SBEA from FRED for annualised one-quarter growth. See the not for publication appendix.

[2] The Federal Reserve Bank of Chicago NFCI data are available for download from FRED.

[3] Related research on inflation at risk includes Korobilis (2017), Ghysels, Iania and Striaukas (2018), Lopez-Salido and Loria (2020) and Banerjee, Contreras, Mehrotra and Zampolli (2020).

ables (including the constant). The predicted value is the quantile for $y_{t+1}$ conditional on $x_t$, $\hat{\mathbf{Q}}_{y_{t+1}|x_t}(\tau|x_t) = x_t\hat{\beta}_\tau$.

## 2.1 Copula-based Methods

Although conventional QR is generally more robust to extreme observations than least squares, the recent pandemic data provide a severe challenge. Furthermore, the conventional QR coefficients are influenced by distributional features (for example, skew) as well as dependence.

Copula-based methods seperate dependence from distributional features. Some recent copula studies in macroeconomics, including Smith (2015), Smith and Vahey (2016) and Loaiza-Maya and Smith (2019), fit non-parametric marginals and parametric copula distribution functions.[4]

Fully non-parametric empirical copula studies typically convert each variable to rank data which are uniformly distributed on the unit interval prior to fitting dependence.[5] The underlying idea is that ranks provide a robust route to assessing dependence that is independent of the marginal distribution of each variable concerned. Whereas, for example, Amengual, Sentana and Tian (2020) consider an implicit linear regression copula function, we consider an implicit QR copula function. Specifically, a QR of economic growth on lagged financial conditions, with lagged economic growth as a further conditioning variable, using pseudo data which are derived from the rank data.

More formally, for each macroeconomic variable in ABG's model space, contained in $y_{t+1}$ and $x_t$ (excluding the intercept), define the rank data as $y^r_{t+1} = R_{y,t+1}/(T+1)$ and $x^r_t = R_{x,t}/(T+1)$, where (for example) $R_{y,t+1}$ denotes the rank of each observation for $y_{t+1}$ relative to its own history. The length of each time series variable is $T$; the $T+1$ denominator avoids boundary issues.

The rank data variables, $y^r_{t+1}$ and $x^r_t$, are converted to be (individually) Gaussian distributed using the inverse Gaussian Cumulative Distribution Function (CDF) to give the "pseudo data"

[4]Copula studies considering the relationship between economic growth and financial conditions include Karagedikli, Vahey and Wakerly (2019), using a parametric combination of non-linear experts, and Coe and Vahey (2020), deploying a fully non-parametric empirical copula model.

[5]Examples of empirical copula studies include Deheuvals (1979) and Velasquez-Giraldo et al (2018).

variables, $y^p_{t+1}$ and $x^p_t$, respectively. QR on these pseudo data selects parameters to minimise the quantile weighted absolute value of errors—the pseudo data analogue to conventional QR. Hereafter, we refer to this implicit copula approach based on pseudo data as the "copula QR" methodology, which we compare and contrast with conventional QR (as used by ABG).

The copula QR approach selects parameters to minimise the quantile weighted absolute value of errors in the pseudo data. That is,

$$\hat{\beta}^p_\tau \;=\; \underset{\beta^p_\tau \in \mathbb{R}^k}{\arg\min} \sum_{t=1}^{T-1} (\tau \cdot \mathbf{I}_{(y^p_{t+1} \geq x^p_t \beta^p)} |y^p_{t+1} - x^p_t \beta^p_\tau| + (1-\tau) \cdot \mathbf{I}_{(y^p_{t+1} < x^p_t \beta^p)} |y^p_{t+1} - x^p_t \beta^p_\tau|),$$

where the predicted value for the pseudo data is the quantile for $y^p_{t+1}$ conditional on $x^p_t$, $\hat{\mathbf{Q}}^p_{y^p_{t+1}|x^p_t}(\tau|x^p_t) = x^p_t \hat{\beta}^p_\tau$. Predictions matched to the scale of the observed data utilise the Gaussian CDF and then the inverse Empirical CDF (ECDF) for each variable individually.[6]

# 3 Results

We break this section into three parts. First, we consider the narrow replication of ABG's findings with their methods and data. Second, we consider the extended sample with conventional QR.[7] Third, we consider the extended sample with copula QR methods.

Throughout, we focus on the multivariate relationship between the one-quarter measure of economic growth and the (lagged) NFCI measure of financial conditions, using lagged economic growth as an additional conditioning variable, to match ABG's approach.[8]

[6] See the application for details of ECDF fitting. Smith and Vahey (2016) consider skew-t marginal distributions for macroeconomic data but find the non-parametric approach used here more effective.

[7] The not for publication appendix includes results using ABG's methods with the extended sample and includes conditional forecast densities for economic growth based on interpolation with the skew-t distribution.

[8] We report results based on a four-quarter measure of economic growth in the not for publication appendix.

## 3.1 Replication with ABG's Sample

Figure 2 plots the conventional QR lines, for the $5^{th}$ through $95^{th}$ percentiles, estimated using ABG's quarterly sample from 1973:1 to 2015:4.[9]. The regression lines are overlaid with a scatterplot of ABG's sample for lagged NFCI and the one-quarter measure of economic growth, where the data have been standardised.[10] Our conventional QR lines displayed in Figure 2 are based on a multivariate QR, evaluated at mean lagged economic growth.

Despite the data revisions since ABG's analysis, Figure 2 is consistent with ABG's findings.[11] In particular, the slopes of the QR lines vary across $\tau$, with the $33^{rd}$ through to the $67^{th}$ percentiles displaying the negative relationship between economic growth and financial conditions described by ABG. In contrast, as ABG note, the $90^{th}$ and $95^{th}$ percentiles, which are imprecisely estimated, are relatively unresponsive to financial conditions. The scatterplot of ABG's (standardised) sample, shown in red, illustrates the non-Gaussian features apparent for both variables. There is a pile up of observations for low NFCI values and a long sparse tail for high NFCI values. The (unconditional) distribution for economic growth has a long left tail as noted by ABG.[12]

The potential for heteroskedasticity has been noted by (among others) Loaiza-Maya and Smith (2020) and Carriero, Clark and Marcellino (2020). We consider the test of Machado and Santos Silva (2000, MSS) as a check for the relevance of QR methods for particular percentiles. Based on ocular inspection of the scatterplots, and the discussion of heteroskedasticity by ABG, we assume that the heteroskedasticity is linearly related to financial conditions.[13] The null hypothesis of homoskedasticity is rejected at the 5% significance level from $\tau = 0.33, \dots, 0.67$. Hence, the tests confirm that the dispersion of the data are related to financial conditions—consistent with the fan-shaped conventional QR lines presented by ABG and in Figure 2.

[9] ABG's conventional bivariate QR plots use $\tau = 0.05, 0.5$ and $0.95$; see, ABG's Figure 3. ABG also display their estimates of the conventional multivariate QR slope coefficients for $\tau = 0.01, \dots, 0.99$ in their Figure 4.

[10] We standardised the data, prior to estimation, to be mean 0 and standard deviation 1. We report confidence bands for the QR coefficients in the appendix.

[11] Our data were downloaded on February $10^{th}$, 2021. See the not for publication appendix for data details.

[12] See the subsequent discussion of Figure 6 on fitting the unconditional distribution.

[13] See the not for publication appendix for details of the two-tailed MSS test and a table of results.

## 3.2 Replication with the Extended Sample

Figure 3 plots the conventional QR lines using the extended quarterly sample from 1973:1 to 2020:4. To facilitate comparison with Figure 2, the axes match and, as before, the conventional QR lines are overlaid with the scatterplot of the (standardised extended) sample. The scatterplot excludes the two recent extreme observations for 2020:2 and 2020:3, shown in Figure 1, which are so far from the unconditional mean of economic growth that they are beyond the limits of the y-axis.[14] As for Figure 2, the regression lines displayed in Figure 3 are based on a conventional multivariate QR, evaluated at mean lagged economic growth.

Based on the full extended sample up to 2020:4, the estimated QR slopes display variation across percentiles in Figure 3, exhibiting the familiar fan-shaped pattern. For the $33^{rd}$ through to the $67^{th}$ percentiles, there is again a negative relationship between the variables. However, the slopes of the conventional QR lines for these $\tau$ values are somewhat attenuated relative to those shown in Figure 2. For example, the point estimates of the slope coefficients for $\tau = 0.33, 0.5$ and 0.67 are -0.440, -0.301 and -0.268, respectively, based on ABG's sample (Figure 2). But, for the extended sample, (Figure 3) the corresponding slopes are -0.382, -0.205 and -0.186.[15]

We note that the conventional QR lines for right tail, for example, the $90^{th}$ and $95^{th}$ percentiles, are relatively flat in both Figure 2 and Figure 3. There is limited evidence of upper tail risk with conventional QR on the extended sample—consistent with ABG's analysis.

Turning to the MSS test, the null hypothesis of homoskedasticity is rejected at the 5% significance level from $\tau = 0.33, \ldots, 0.67$, which confirms our earlier finding with ABG's sample.

## 3.3 Copula Methods with the Extended Sample

Since the extended sample displays minor attenuation in the relationship between economic growth and financial conditions, we reassess the evidence exploiting copula QR methods. Recall

[14] To match our prior analysis of ABG's sample, we standardised the extended sample data (prior to estimation) to be mean 0 and standard deviation 1. Confidence bands are reported in the not for publication appendix.

[15] Results for both samples are provided in the not for publication appendix.

that our approach utilises a distribution-robust framework for fitting dependence, suitable for analysis in the presence of extreme observations.

Figure 4 plots the regression lines from the copula QR, using the quarterly sample from 1973:1 to 2020:4, overlaid with a scatterplot for the extended sample (in blue). In this case, the scatterplot displays the pseudo data (derived from the rank data).[16] The copula QR parameters were estimated on the (multivariate vector of) pseudo observations.

The copula QR fitted slope coefficients for the extended sample exhibit the familiar pattern across the $33^{rd}$ and $67^{th}$ percentiles—broadly consistent with ABG's findings and their fan-shaped QR lines. For example, the copula QR slope coefficients for $\tau = 0.33, 0.5$ and $0.67$ are -0.349, -0.272 and -0.220, respectively.[17] For the (relatively) imprecisely estimated $90^{th}$ and $95^{th}$ percentiles, the copula QR slopes are slightly positive on the extended sample. Turning to the MSS test, for the pseudo extended sample, the null hypothesis of homoskedasticity is rejected at the 5% significance level from $\tau = 0.33, \ldots, 0.67$. In summary, the copula QR analysis is consistent with ABG's analysis of their sample ending in 2015:4.

To illustrate the implications of the copula QR methodology for observed economic growth and (lagged) financial conditions, Figure 5 displays the corresponding non-linear copula QR lines, overlaid with the extended (standardised) sample. The lines plotted in Figure 5 exploit the Gaussian CDF and then the (inverse) ECDF for each variable. The ECDFs were fitted with the SSV locally adaptive kernel density estimator by Shimazaki and Shinomoto (2010). Figure 6 displays the (unconditional) density for economic growth based on the extended sample, in red, together with the equivalent density for ABG's sample, in black. Influenced by the recent extreme observations, the latter is considerably more diffuse than the former.[18]

The modest extent of the non-linearity in Figure 5 can be judged from the variation in

[16] Including the extreme observations from 2020:2 and 2020:3 which are within the range of the axes.

[17] Pseudo sample fitted coefficients are provided in the not for publication appendix, along with confidence bands, for both ABG's sample and the extended sample. The corresponding fitted coefficients for ABG's sample are -0.348, -0.302 and -0.267, respectively.

[18] See the not for publication appendix for tests of non-normality based on the unconditional distributions.

the slopes of the copula QR lines across NFCI values. For example, for the $33^{rd}$ through $67^{th}$ percentiles, the copula QR lines are fairly steep near the central mass of the scatterplot. The lines flatten somewhat for NFCI just above the mode, then steepen again, and then flatten again, moving into the right tail for NFCI. Nevertheless, the negative relationship between economic growth and (lagged) financial conditions is apparent, and given the familiar fan-shaped pattern, the copula QR lines are broadly consistent with ABG's analysis of their sample.[19]

To illustrate the robustness to the distributional issues arising from the recent observations, Figure 7 displays the counterpart to Figure 5 using ABG's sample (ending in 2015:4). The copula QR lines for the $33^{rd}$ through $67^{th}$ percentiles vary little by sample.

# 4 Conclusions

ABG utilise QR methods in their analysis of the relationship between U.S. economic growth and (lagged) financial conditions. We replicate their main economic findings in terms of dependence using both their sample and an extended sample (including the 2020 pandemic observations). We find that the distribution-robust dependence in the extended sample resembles ABG's characterisation based on pre-pandemic data.

[19]The not for publication appendix shows that the (conditional) predictive densities from the model exhibit scope for multi-modality consistent with Adrian, Boyarchenko and Giannone (2019b).

**Figure 1: One-quarter Economic Growth and Lagged Financial Conditions, Extended Sample**

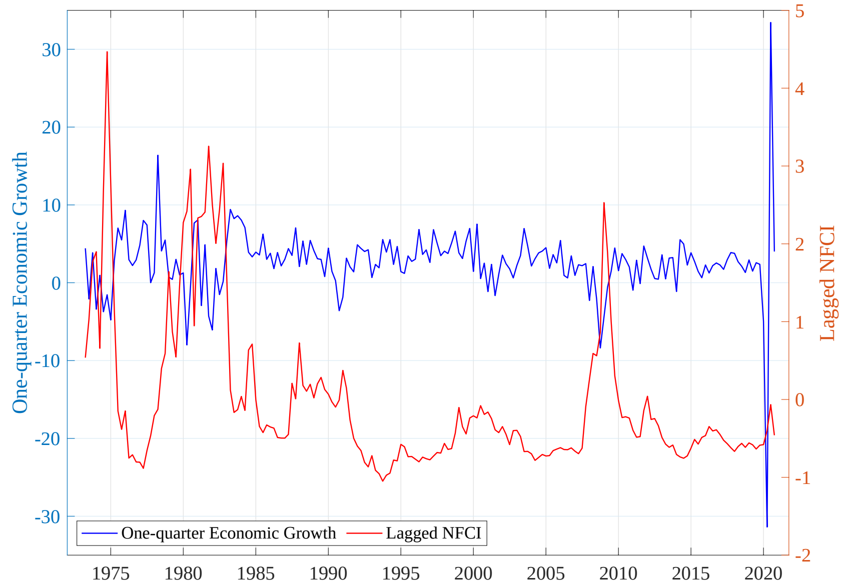
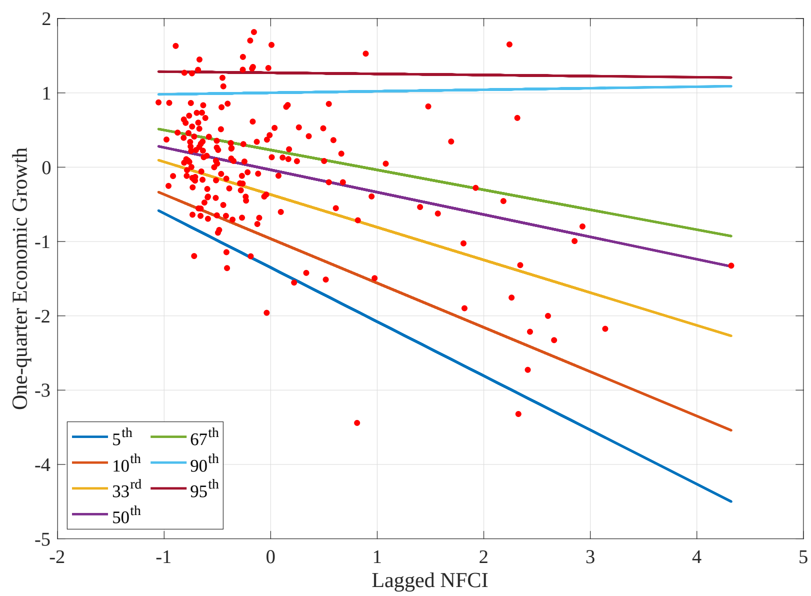
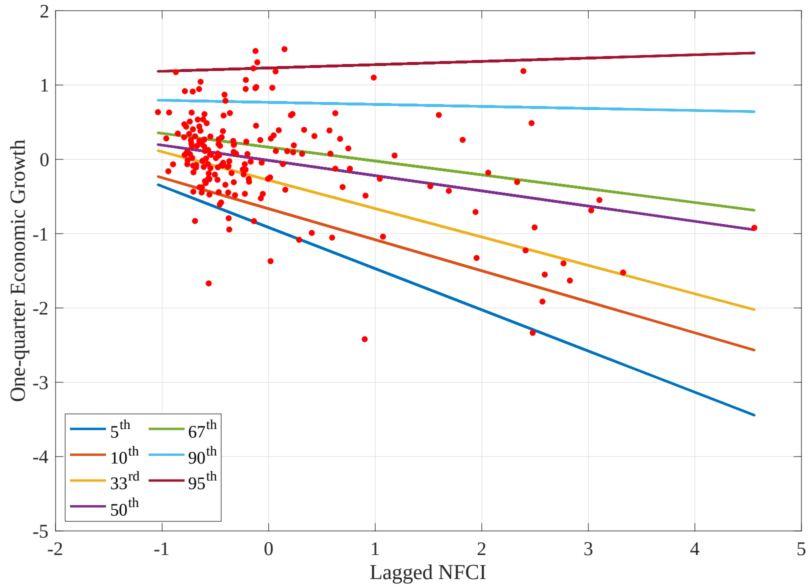



**Figure 2: Conventional QR Lines, ABG's Sample**



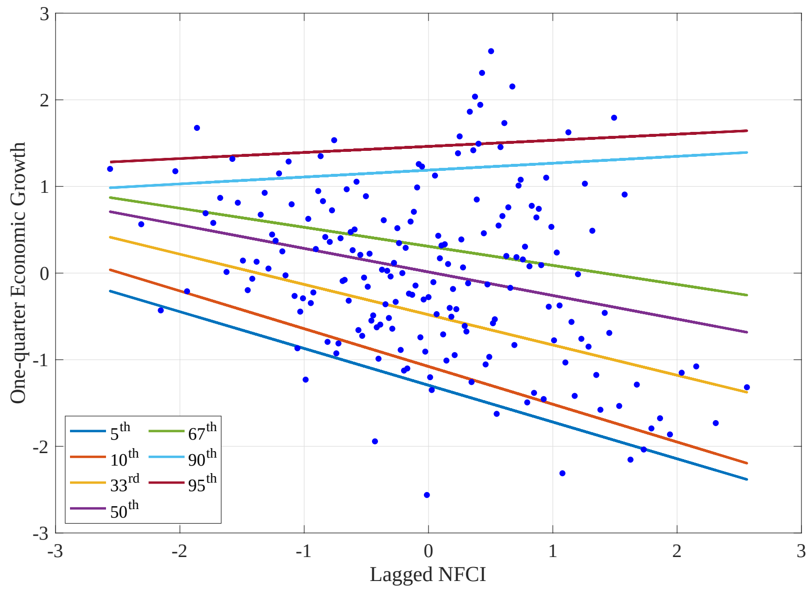

Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots).

## Figure 3: Conventional QR Lines, Extended Sample



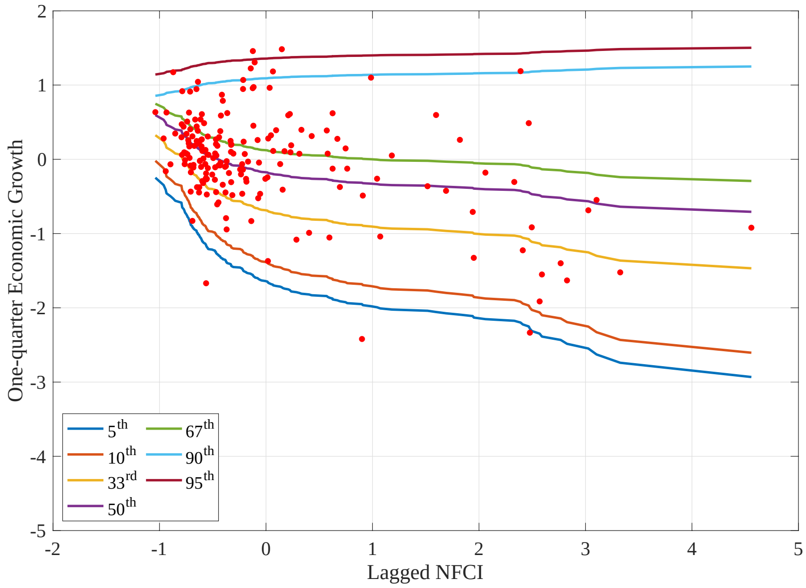

Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots).

## Figure 4: Copula QR Lines, Extended Pseudo Sample



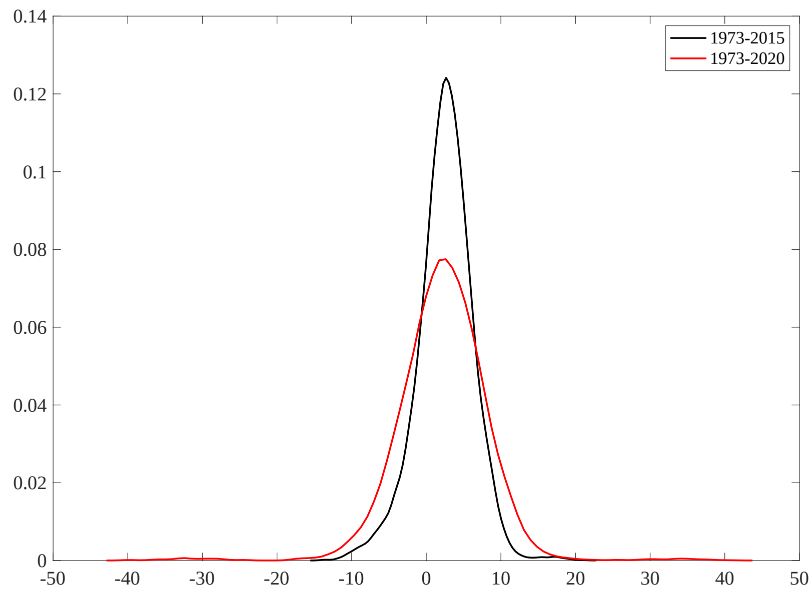

Note: QR lines overlaid with a scatterplot of the pseudo data (blue dots).

## Figure 5: Non-linear Copula QR Lines, Extended Sample



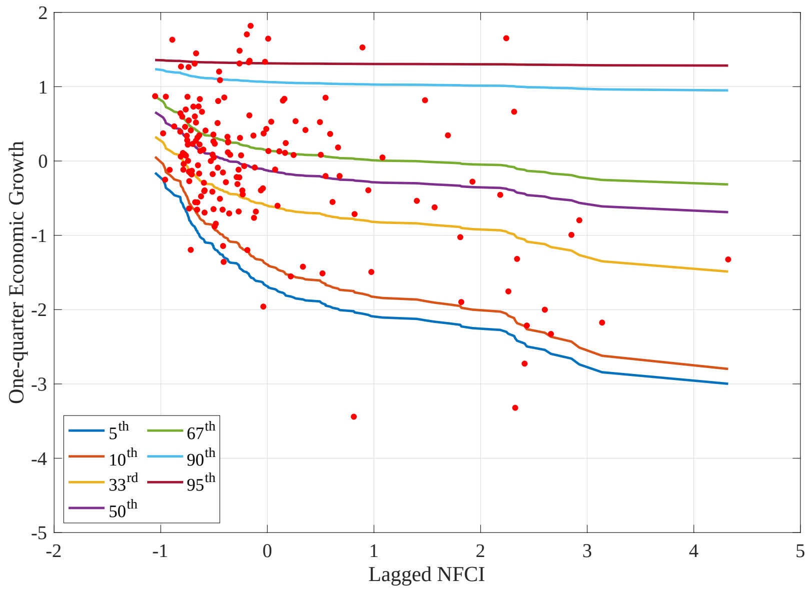

Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots).

## Figure 6: Marginal Densities for One-quarter Economic Growth, ABG's and Extended Sample




Note: Marginal densities fitted using the SSV method of Shimazaki and Shinomoto (2010).

## Figure 7: Non-linear Copula QR Lines, ABG's Sample




Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots).

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