Reassessing the Dependence Between Economic Growth and Financial Conditions since 1973*

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Abstract

Adrian, Boyarchenko and Giannone (2019, ABG) adapt Quantile Regression (QR) methods to examine the relationship between U.S. economic growth and financial conditions. We confirm their empirical findings, using their methodology and their pre-2016 sample. Mindful of the importance of the Covid-19 pandemic, we extend the sample to 2021:3 and find attenuation of the key estimated coefficients using ABG's empirical methods. Given the pandemic observations, we provide robust QR analysis of dependence based on ranked data, and explain the relationship with extant copula modelling methods.

Keywords: Vulnerable Growth, Quantile Regression, Copula Modelling

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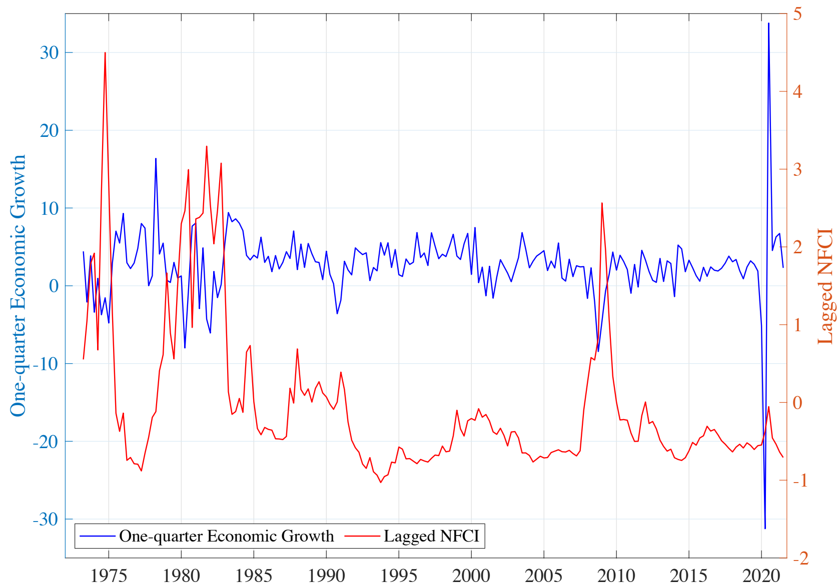
1 Introduction

This paper reassesses the dependence between economic growth in the United States and financial conditions studied by Adrian, Boyarchenko and Giannone (2019). Figure 1 plots the quarterly data using a one-quarter annualised measure of economic growth and the National Financial Conditions Index (produced by the Chicago Fed). Notable features since the turn of the century include: the negative co-movements during the Global Financial Crisis (GFC); the relative stability of both variables during the post-GFC recovery; the sharp drop in economic growth for 2020:2 to (around) -30%, followed by the immediate rebound in economic growth for 2020:3; and, the relative stability of both variables since 2020:3.

Figure 1: One-quarter Economic Growth and Lagged Financial Conditions, Extended Sample



ABG argue that economic growth and financial conditions exhibit a negative relationship utilising Quantile Regression (QR) methods, particularly in the left tail of the output growth distribution. Based on evidence up to 2015:4, ABG's sample excludes the last 23 quarterly observations displayed in Figure 1, and in particular, those associated with the Covid-19 pandemic.

In the subsequent sections of this paper, we summarise ABG's approach and reconsider the evidence using ABG's sample and the extended sample. For both sample end dates, we compare ABG's conventional QR and a copula QR methodology. When considering the additional observations in the extended sample, the key coefficients estimated via conventional QR exhibit attenuation. In contrast, the more robust copula QR approach reveals negligible attenuation in dependence.

Whereas ABG's conventional QR approach does not separate marginal distributions from dependence, the copula variant of QR does. The copula approach uses ranked data, which helps mitigate the influence of outliers. The (non-parametric) transform to ranks removes the cardinal influence of any outliers but preserves the ordinal multivariate dependence between the macroeconomic variables. To avoid issues that arise when fitting dependence to data on the unit interval, we convert the ranked data for each variable to be individually Gaussian distributed using a positive monotone transform. Then, we use QR methods to fit the linear dependence between the Gaussian-distributed "pseudo" variables, allowing for heteroskedasticity.

2 ABG's Data, Model Space and Quantile Regressions

ABG consider the relationship between U.S. economic growth and lagged financial conditions, including lagged economic growth as a conditioning variable. Using a sample from 1973:1 to 2015:4, they analyse a variety of measures for both economic growth and financial conditions. Their main results focus on the relationship between one-quarter (and four-quarter) economic growth and the (lagged) National Financial Conditions Index (NFCI).[1] The NFCI is a weekly factor model based index of financial conditions derived from over 100 indicators from money, debt and equity markets. Positive values for the NFCI imply that financial conditions are tighter than average by construction.

ABG's empirical contribution comprises two main elements. First, their analysis of dependence with the conditional quantile function using conventional QR. Their analysis suggests a negative relationship with the NFCI at the median, with a stronger (weaker) relationship in the lower (upper) tail of economic growth. Second, their analysis of conditional forecast densities for

[1] We describe the annualised economic growth and NFCI data from FRED in the online appendix.

economic growth based on interpolation with the skew-t distribution (Azzalini and Capitanio, 2003). In this paper, we focus on their analysis of dependence with conventional QR.[2]

Recent macroeconomic studies using (modified) QR methods include: Giglio, Kelly, and Pruitt (2016), De Nicolo and Lucchetta (2017), Chavleishvili and Manganelli (2019), Ferrara, Mogliani, and Sahuc (2021) and Loria, Matthes and Zhang (2022). The QR approach also underlies the IMF's bilateral surveillance tool known as "growth at risk" (Adrian et al., 2018).[3] Reichlin, Ricco, and Hasenzagl (2020) examine alternative indicators for assessing the risks to economic growth. Carriero, Clark and Marcellino (2020) argue that stochastic volatility models provide a robust alternative to QR for quantifying these risks.

Following ABG, we denote the target variable, economic growth in $t+1$ as y_{t+1}, and the explanatory variable vector, comprising a constant, economic growth and NFCI in t, as x_t. ABG assume that the relationship between the target and the explanatory variable is linear for each quantile and select parameters to minimise the quantile weighted absolute value of errors. That is,

$$\hat{\beta}_\tau \ = \ \underset{\beta_\tau \in \mathbf{R}^k}{\arg\min} \sum_{t=1}^{T-1} (\tau \cdot \mathbf{I}_{(y_{t+1} \geq x_t \beta)} |y_{t+1} - x_t \beta_\tau| + (1-\tau) \cdot \mathbf{I}_{(y_{t+1} < x_t \beta)} |y_{t+1} - x_t \beta_\tau|),$$

where $I(\cdot)$ denotes the indicator function and k denotes the number of explanatory variables (including the constant). The predicted value is the quantile for y_{t+1} conditional on x_t, $\hat{\mathbf{Q}}_{y_{t+1}|x_t}(\tau|x_t) = x_t \hat{\beta}_\tau$.

It is well known that conventional QR makes no a priori assumptions about the marginal distributions of the variables concerned.

2.1 Copula-based Methods

Although conventional QR is generally more robust than least squares, the recent pandemic data provide a severe challenge. Copula-based methods fit dependence based on uniformly-distributed

[2]We consider forecast densities in the online appendix.

[3]Related research on inflation risks includes Korobilis (2017), Ghysels, Iania and Striaukas (2018), Lopez-Salido and Loria (2020) and Banerjee, Contreras, Mehrotra and Zampolli (2020).

data on the unit interval, which contributes to robustness in the presence of outliers, as Amengual, Sentana and Tian (2020) note. Some recent macroeconomic copula studies, including Smith (2015), Smith and Vahey (2016) and Loaiza-Maya and Smith (2020), fit parametric copula distribution functions, with the last two explicitly considering the risks to economic growth in the U.S.[4] In contrast, fully non-parametric copulas, sometimes known as "empirical copulas", fit the dependence non-parametrically to ranked data; see, among others, Deheuvals (1979) and Velasquez-Giraldo et al (2018). Coe and Vahey (2020) deploy a fully non-parametric empirical copula model to assess the relationship between economic growth and financial conditions using historical data.

Given the potential importance of the pandemic observations, we consider a robust variant of QR based on transformations for ranked data. Specifically, we transform the ranked data for each variable to construct individually Gaussian-distributed variables. And, then deploy QR to assess the dependence between the variables.

The implicit copula approach adopted in Smith (2015), Smith and Vahey (2016) and Loaiza-Maya and Smith (2020) exploits positive monotone transformations of uniformly-distributed data. Other recent studies adopting similar transformations include: the analysis of nonparanormal graphical models by Liu, Lafferty and Wasserman (2009) and Mulgrave and Ghosal (2020); the transnormal (nonparanormal) quantile regression framework for high-dimensional data developed by Fan, Xue and Zou (2016); and, the kernel density estimation methods developed by Wen and Wu (2018).

More formally, for economic growth, y_{t+1}, we construct the (ascending) ranked data as $y^r_{t+1} = R_{y,t+1}/(T + 1)$, where $R_{y,t+1}$ denotes the rank of each observation for y_{t+1} relative to its own history. The length of each time series variable is T. In a similar manner, we also transform to ranks each explanatory variable in x_t (excluding the intercept). Each ranked data variable is then transformed to be (individually) Gaussian using an inverse Gaussian Cumulative Distribution Function (CDF). The approach generates standard Normal "pseudo" data, y^p_{t+1} and x^p_t.

[4] Karagedikli, Vahey and Wakerly (2019) consider a semi-parametric copula combination model.

To see that our copula approach mitigates the influence of outliers, note that the transformation to ranks preserves the ordinal structure of the data but removes the cardinal content and hence the scale of the outliers. Suppose, for example, that the highest value for economic growth is 30%, and second highest is 5%, with a sample standard deviation of 5. Then the gap between the two adjacent ranked observations is 5 standard deviations. In contrast, the ranked data are uniformly distributed on the unit interval, with the two highest observations being $T/(T+1)$ and $(T-1)/(T+1)$.

It is well known that ordinal multivariate dependence is preserved when data are ranked. For computational convenience, we transform the ranked data for each variable to be individually Gaussian distributed to avoid the boundary issues that arise with data defined on the unit interval. These pseudo data preserve the (ordinal) dependence of the ranked data; the (inverse) Gaussian CDF provides the rank-preserving positive monotone transform. In this manner, the cardinal content of the outliers is purged before fitting the dependence.

QR on the pseudo data selects parameters to minimise the quantile weighted absolute value of errors—the pseudo data analogue to conventional QR, which we refer to as "copula QR". That is,

$$\hat{\beta}^p_\tau \;=\; \underset{\beta^p_\tau \in \mathbf{R}^k}{\arg\min} \sum_{t=1}^{T-1} (\tau \cdot \mathbf{I}_{(y^p_{t+1} \geq x^p_t \beta^p)} |y^p_{t+1} - x^p_t \beta^p_\tau| + (1-\tau) \cdot \mathbf{I}_{(y^p_{t+1} < x^p_t \beta^p)} |y^p_{t+1} - x^p_t \beta^p_\tau|),$$

where the predicted value for the pseudo data is the quantile for y^p_{t+1} conditional on x^p_t, $\hat{\mathbf{Q}}^p_{y^p_{t+1}|x^p_t}(\tau|x^p_t) = x^p_t \hat{\beta}^p_\tau$.

In contrast to ABG's conventional QR approach, where dependence is related to the marginal distributions of the variables, the copula QR methodology separates marginal distributions from dependence. Hence, copula QR offers an outlier-robust route to gauging dependence by utilising transformed ranked data.

3 Results

We break this section into three parts. First, we consider the narrow replication of ABG's findings with their methods and data. Second, we consider the extended sample, which includes the pandemic observations, with conventional QR. Finally, we consider the extended sample with copula QR methods. Throughout, we focus on the multivariate relationship considered by ABG, using a one-quarter measure of economic growth.[5]

3.1 Replication with ABG's Sample

Figure 2 plots the conventional QR lines, for $\tau = 5, \ldots, 95$, fitted using ABG's quarterly sample from 1973:1 to 2015:4.[6]. The regression lines are overlaid with a scatterplot of ABG's sample data for lagged NFCI and the one-quarter measure of economic growth.[7] Our regression lines displayed in Figure 2 are based on a multivariate QR, evaluated at mean lagged economic growth.

Despite the data revisions since ABG's analysis, Figure 2 is consistent with their findings. In particular, the slopes of the QR lines vary across τ, displaying the negative relationship between economic growth and financial conditions described by ABG. For low (high) τ, the relationship is stronger (weaker) than at the centre of the economic growth distribution. The scatterplot of ABG's (standardised) sample, shown in red, illustrates the non-Gaussian features apparent for economic growth and the NFCI. Notably, there is a pile up of observations for low NFCI values and a long sparse tail for high NFCI values.

Since the potential for heteroskedasticity has been noted by (among others) Loaiza-Maya and Smith (2020) and Carriero, Clark and Marcellino (2020), we consider the test of Machado and Santos Silva (2000, MSS) to check the relevance of QR methods in this application. Using ocular inspection of the scatterplots, combined with the discussion of heteroskedasticity by ABG, we assume that the heteroskedasticity is linearly related to financial conditions. The null hypothesis

[5] We report results based on a four-quarter measure of economic growth in the online appendix.

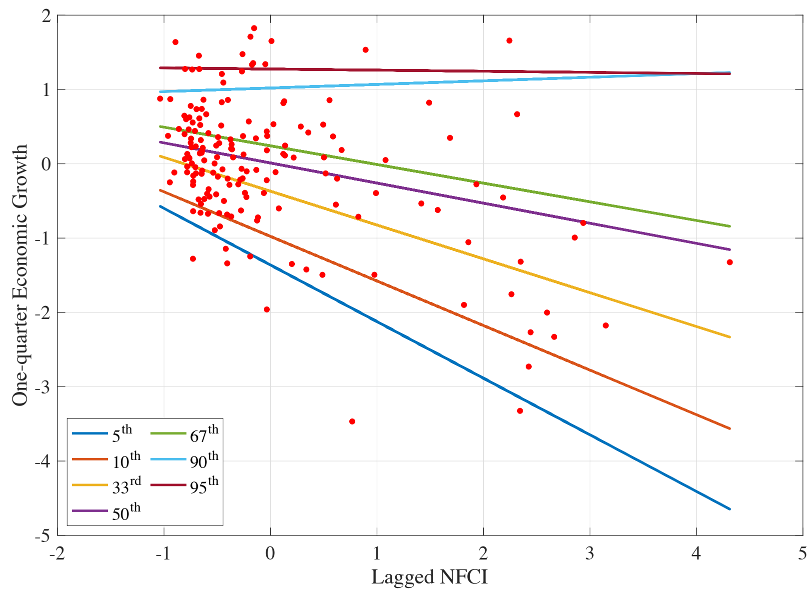
[6] ABG provide bivariate QR plots for $\tau = 0.05, 0.5$ and 0.95 in their Figure 3. They display point estimates of the multivariate QR coefficients for $\tau = 0.05, 0.10, \ldots, 0.95$ in their Figure 4.

[7] We standardised the sample data, prior to estimation, to be mean 0 and standard deviation 1.

of homoskedasticity is rejected at the 5% significance level from $\tau = 0.33, \ldots, 0.67$. Hence, these tests confirm that the fan-shaped conventional QR lines presented by ABG and replicated in our Figure 2 are consistent with heteroskedasticity.[8]

The correlation between economic growth and the NFCI is statistically significant based on a 90% confidence interval, using a moving block bootstrap, for the conventional QR lines corresponding to $\tau = 0.33, 0.5, 0.67$.[9] We describe our bootstrap methodology in the online appendix, and provide results in the upper panel of Table A4.

Figure 2: Conventional QR Lines, ABG's Sample



Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots).

3.2 Replication with the Extended Sample

Turning to our analysis with the extended sample, 1973:1 to 2021:3, Figure 3 plots the conventional QR lines. The axes match Figure 2 but the QR lines in Figure 3 are overlaid with the scatterplot of the (standardised) extended sample data. The scatterplot excludes the two recent extreme observations for 2020:2 and 2020:3, which are beyond the limits of the y-axis.

[8]We present these heteroskedasticity test results in Table A2 of the online appendix.

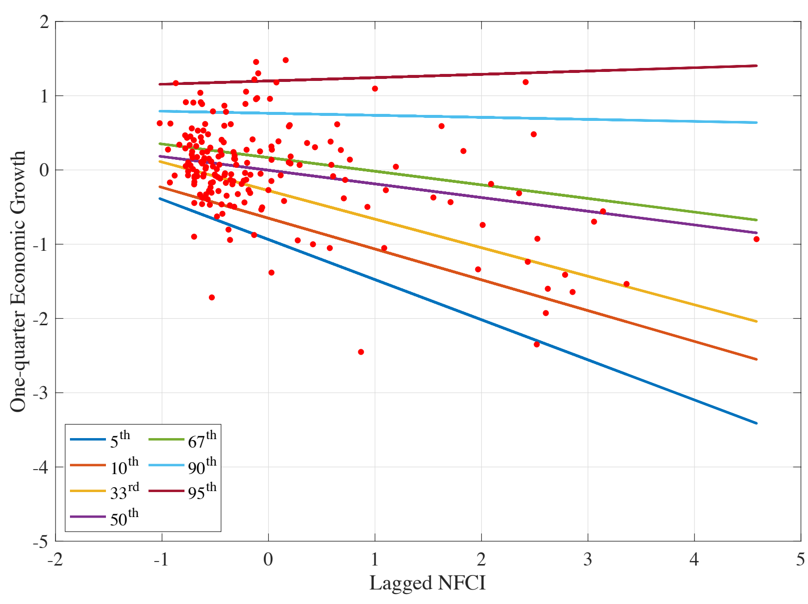
[9]ABG's Figure 4 compares the QR estimates with (bootstrapped) confidence intervals for the parameters of a (linear, homoskedastic, Gaussian) vector autoregression. We present a similar plot in the online appendix.

The QR slopes displayed in Figure 3 vary with τ, exhibiting the familiar fan-shaped pattern noted by ABG. But, the estimated coefficients are attenuated for the extended sample that includes the pandemic observations. For example, the point estimates of the slope coefficients for $\tau = 0.33, 0.5$ and 0.67 are -0.455, -0.270 and -0.251, respectively, based on ABG's sample (Figure 2). However, for the extended sample (Figure 3), the corresponding slopes are -0.384, -0.184 and -0.183.

Turning to the MSS test based on the extended sample, the null hypothesis of homoskedasticity is rejected at the 5% significance level from $\tau = 0.33, \ldots, 0.67$—consistent with our earlier finding of heteroskedasticity with ABG's sample.[10]

We confirmed that the relationship between economic growth and the NFCI is statistically significant based on a 90% confidence interval, using a moving block bootstrap, for the conventional QR lines corresponding to $\tau = 0.33, 0.5, 0.67$. We present results in the lower panel of Table A4 in the online appendix.

Figure 3: Conventional QR Lines, Extended Sample



Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots).

Overall, we conclude that the relationship between economic growth and financial conditions is somewhat weaker for the extended sample. To investigate this issue further, we conducted

[10]We present these heteroskedasticity test results in Table A2 of the online appendix.

a Monte Carlo simulation of the impact of outliers matched to the marginal distribution of the post-2015 sample. In our simulations based on 10,000 samples of length 200, the average attenuation in the conventional QR coefficients is around 30% at the median; see, Section A.2.4 of the online appendix.

As a further robustness check, we repeated our analysis including a dummy variable for the pandemic observations, and still found attenuation in the QR coefficients for lagged NFCI.[11]

3.3 Copula Methods with the Extended Sample

Recall that the copula QR approach utilises the information in the ranks to assess dependence, and so mitigates the cardinal impacts of outliers. Turning to our application with the extended sample, Figure 4 plots the copula QR lines, overlaid with a scatterplot of the pseudo data (in blue, derived from the ranked data), including the extreme observations from 2020:2 and 2020:3.

The copula QR lines for the extended sample exhibit the familiar fan-shaped pattern and resemble the copula QR lines based on ABG's sample. For example, the copula QR coefficients for $\tau = 0.33, 0.5$ and 0.67 are -0.334, -0.254 and -0.205, respectively. The copula QR coefficients for ABG's sample are very similar: -0.328, -0.293 and -0.252, respectively.

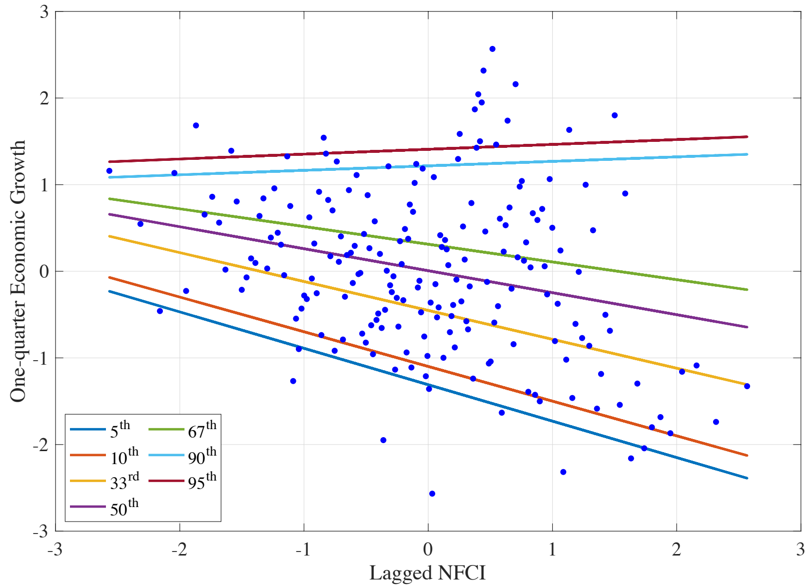
Turning to the MSS test with the pseudo extended sample, the null hypothesis of homoskedasticity is rejected at the 5% significance level from $\tau = 0.33, \ldots, 0.67$—consistent with our earlier findings of heteroskedasticity.[12]

We confirmed that the relationship between economic growth and the NFCI is statistically significant based on a 90% confidence interval, using a moving block bootstrap, for the copula QR lines corresponding to $\tau = 0.33, 0.5, 0.67$. We present results in the lower panel of Table A5 in the online appendix.[13]

In summary, the copula QR approach reveals a robust relationship with between output growth and (lagged) NFCI based on the extended sample including the pandemic observations.

[11] We present results including consideration of a pandemic dummy variable in Table A6 of the online appendix.
[12] We present heteroskedasticity test results in Table A3 of the online appendix.
[13] We present copula QR confidence intervals for ABG's sample in the upper panel of Table A5 of the online appendix.

Figure 4: Copula QR Lines, Extended Pseudo Sample



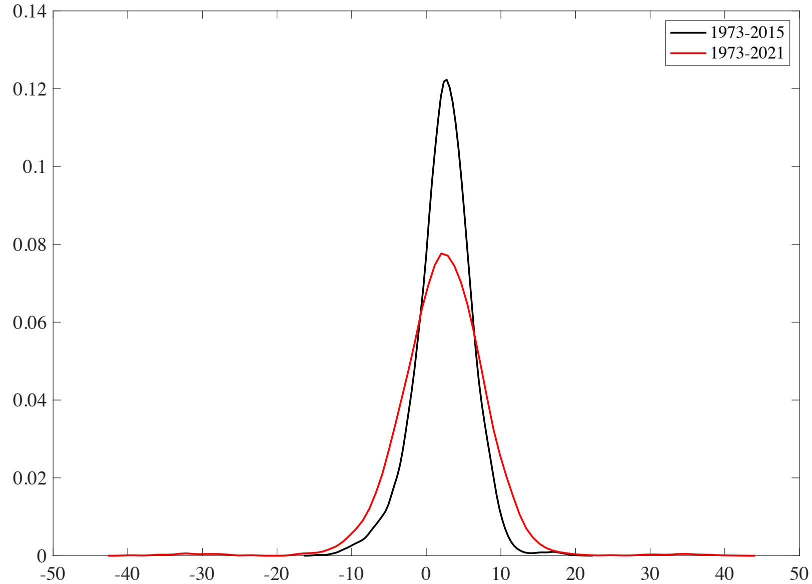
Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots).

We emphasise that copula QR dependence is linear, as displayed in Figure 4. Nevertheless, with non-Gaussian marginal distributions, there is scope for non-linearities. To illustrate the impact of non-Gaussian marginal distributions, we fit the Empirical CDFs for each variable using the SSV locally adaptive kernel density estimator of Shimazaki and Shinomoto (2010). Figure 5 plots the resulting densities for output growth (only), with the densities for ABG's sample and the extended sample displayed in black and red, respectively.[14] Of course, the latter is considerably more diffuse than the former as a result of the pandemic observations.

The copula QR lines displayed in Figure 6 reflect the (modest) non-linearities introduced by the fitted marginal distributions for both variables. The lines exhibit the familiar fan-shaped pattern and also the generally negative correspondence between the two variables. In this sense, the non-linear copula QR lines are broadly consistent with ABG's conventional QR analysis based on their original sample. That said, most of the non-linear copula QR lines flatten for NFCI values in the range 1 to 2, and again beyond 3.

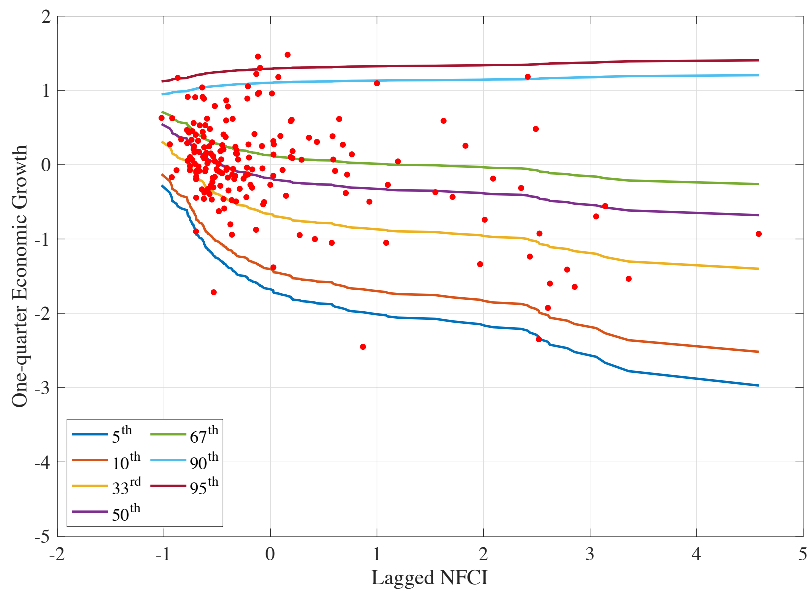
[14]The marginal distributions for the NFCI were fitted using the same kernel density estimator.

Figure 5: Marginal Densities for One-quarter Economic Growth, ABG's and Extended Sample



Note: Marginal densities fitted using the SSV method of Shimazaki and Shinomoto (2010).

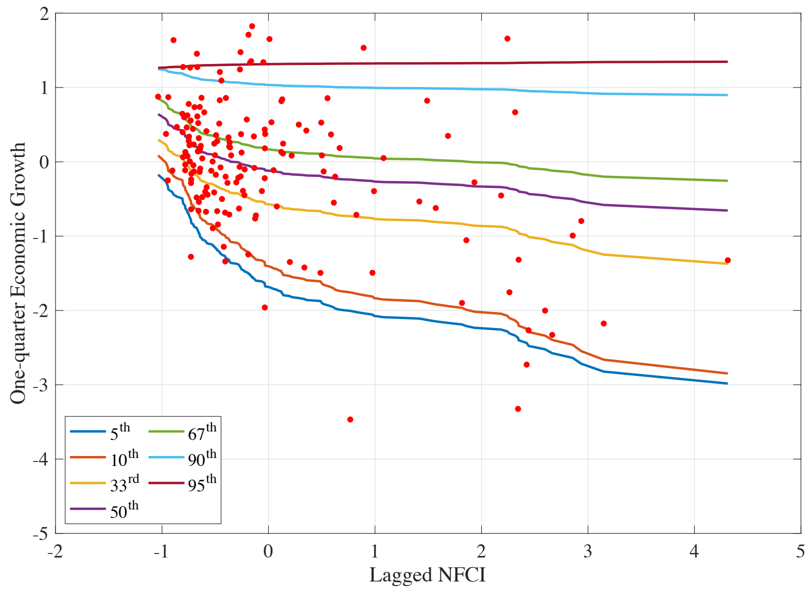
Figure 6: Non-linear Copula QR Lines, Extended Sample



Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots).

As a further check on the robustness of the copula QR approach, Figure 7 displays the non-linear lines for ABG's sample. These are very similar to those for the extended sample displayed in Figure 6.

Figure 7: Non-linear Copula QR Lines, ABG's Sample



Note: QR lines overlaid with a scatterplot of the (standardised) data (red dots).

4 Conclusions

ABG provide conventional QR analysis of the relationship between economic growth and (lagged) financial conditions. We replicate their main economic findings in terms of dependence using both their sample and an extended sample. We find that the fitted linear dependence using copula QR methods including the pandemic observations broadly resembles ABG's original characterisation.

References

Adrian, Tobias, Nina Boyarchenko, and Domenico Giannone (2019) "Vulnerable Growth" *American Economic Review* 109(4) pp1263-1289.

Adrian, Tobias, Nina Boyarchenko, and Domenico Giannone (2021) "Multimodality in Macrofinancial Dynamics" *International Economic Review* 62(2) pp861-886.

Adrian, Tobias, Federico Grinberg, Nellie Liang, and Sheheryar Malik (2018) "The Term Structure of Growth-at-Risk" IMF Working Paper WP/18/180.

Amengual, Dante, Enrique Sentana and Zhanyuan Tian (2020) "Gaussian Rank Correlation and Regression" CEPR Discussion Papers No 14914 (revised July 2020 and forthcoming in A. Chudik, C. Hsiao and A. Timmermann (eds.) "Essays in Honor of M. Hashem Pesaran", Advances in Econometrics, Emerald).

Azzalini, Adelchi, and Antonella Capitanio (2003) "Distributions Generated by Perturbation of Symmetry with Emphasis on a Multivariate Skew t-distribution" *Journal of the Royal Statistical Society Series B (Statistical Methodology)* 65 pp367-389.

Banerjee, Ryan Niladri, Juan Contreras, Aaron Mehrotra and Fabrizio Zampolli (2020) "Inflation at Risk in Advanced and Emerging Market Economies" BIS Working Papers No 883.

Carriero, Andrea, Todd E. Clark and Massimiliano Marcellino (2020) "Capturing Macroeconomic Tail Risks with Bayesian Vector Autoregressions" Federal Reserve Bank of Cleveland Working Paper No. 20-02.

Chavleishvili, Sulkhan and Simone Manganelli (2019) "Forecasting and Stress Testing with Quantile Vector Autoregression" ECB Working Paper No. 2330.

Coe, Patrick J. and Shaun P. Vahey (2020) "Financial Conditions and the Risks to Economic Growth in the United States Since 1875" CAMA Working Paper No. 36/2020.

De Nicolò, Gianni and Marcella Lucchetta (2017) "Forecasting Tail Risks" *Journal of Applied Econometrics* 32(1) pp159-170.

Deheuvels, Paul (1979) "La Fonction de Dépendence Empirique et Ses Propriétés. Un Test non Paramétrique d'Indépendence" *Bulletin Royal Belge de l'Académie des Sciences* 65 pp274-292.

Fan, Jianqing, Lingzhou Xue and Hui Zou (2016) "Multitask Quantile Regression Under the Transnormal Model" *Journal of the American Statistical Association* 111(516) pp1726-1735.

Ferrara, Laurent, Matteo Mogliani and Jean-Guillaume Sahuc (2021) "High-frequency Monitoring of Growth at Risk" *International Journal of Forecasting* available online 6 August 2021.

Ghysels, Eric, Leonardo Iania and Jonas Striaukas (2018) "Quantile-based Inflation Risk Models" Working Paper Research 349 National Bank of Belgium.

Giglio, Stefano, Bryan Kelly, and Seth Pruitt (2016) "Systemic Risk and the Macroeconomy: An Empirical Evaluation" *Journal of Financial Economics* 119 pp457-471.

Karagedikli, Ozer, Shaun P. Vahey and Elizabeth C. Wakerly (2019) "Improved Methods for Combining Point Forecasts for an Asymmetrically Distributed Variable" CAMA Working Paper No. 15/2019.

Koenker, Roger (2005) *Quantile Regression*, Cambridge University Press, Cambridge.

Korobilis, Dimitris (2017) "Quantile Regression Forecasts of Inflation under Model Uncertainty" *International Journal of Forecasting* 33(1) pp11-20.

Liu, H., Lafferty, J. D., and Wasserman, L. A. (2009) "The Nonparanormal: Semiparametric Estimation of High Dimensional Undirected Graphs" *Journal of Machine Learning Research* 10 pp2295-2328.

Loaiza-Maya, Ruben and Michael S. Smith (2020) "Real-Time Macroeconomic Forecasting with a Heteroskedastic Inversion Copula" *Journal of Business and Economic Statistics* 38(2) pp470-486.

Lopez-Salido, J. David, and Loria Francesca (2020) "Inflation at Risk" FEDS Board of Governors of the Federal Reserve System 2020-013.

Loria, Francesca, Christian Matthes, and Donghai Zhang (2022) "Assessing Macroeconomic Tail Risk" FEDS Board of Governors of the Federal Reserve System 2019-026 (revised January 6, 2022).

Machado, J.A.F. and J.M.C. Santos Silva (2000) "Glejser's Test Revisited" *Journal of Econometrics* 97 pp189-202.

Mulgrave, Jami J. and Subhashis Ghosal (2020) "Bayesian Inference in Nonparanormal Graphical Models" *Bayesian Analysis* 15(2) pp449-475.

Reichlin, Lucrezia, Giovanni Ricco and Thomas Hasenzagl (2020) "Financial Variables as Predictors of Real Growth Vulnerability" Deutsche Bundesbank Discussion Papers 05/2020.

Shimazaki, Hideaki and Shigeru Shinomoto (2010) "Kernel Bandwidth Optimization in Spike Rate Estimation" *Journal of Computational Neuroscience* 29(1-2) pp171-182.

Smith, Michael S. (2015) "Copula Modelling of Dependence in Multivariate Time Series" *International Journal of Forecasting* 31(3) pp815-833.

Smith, Michael S. and Shaun P. Vahey (2016) “Asymmetric Forecast Densities for U.S. Macroeconomic Variables from a Gaussian Copula Model of Cross-Sectional and Serial Dependence” *Journal of Business and Economic Statistics* 34(3) pp416-34.

Velasquez-Giraldo, Mateo, Gustavo Canavire Bacarreza, Kim P. Huynh and David T. Jacho-Chavez (2018) ”Flexible Estimation of Demand Systems: A Copula Approach” *Journal of Applied Econometrics* 33(7) pp1109-1116.

Wen, Kuangyu and Ximing Wu (2018) “Transformation-Kernel Estimation of Copula Densities”, *Journal of Business and Economic Statistics* 38(4) pp1-36.